Monopoles, duality, and QED_3

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Based on: arXiv:2210.12370 with Eric Dupuis and William Witczak-Krempa
and arXiv:2310.08343 with Ning Su
and arXiv:2409.17913 with Zohar Komargodski

Quantum Electrodynamics in 2+1 dimensions

- QED_3 is a $U(1)$ gauge theory in 2+1 dimensions coupled to either N complex scalar or (2-component) fermionic fields.
  - N must be even in fermionic case due to parity anomaly.
- One can also add a Chern-Simons coupling with integer coefficient k.
- When N or k is large, theory flows to CFT in IR [Appelquist, Nash, Wijewardhana '88] , but less known at small N, k bc strongly coupled.
- When N, k small, has both condensed matter realizations, and also simplest example of IR duality without supersymmetry!

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- Answer: Monopole operators are window on strongly coupled physics!
  - Large N, k expansion of their scaling dimensions is surprisingly accurate for ALL N, k, use to answer above questions.

Outline

- Define monopole operators in QED3 and results for their scaling dimensions at large N, k.
- Compare to operators in dual theories for $N = 1$ scalars ($O(2)$ WF dual to $k = 0$, free fermion dual to $k = 1$).
- Combine with conformal bootstrap to argue that scalar $N = 2, k = 0$ theory (Neel-VBS) is tricritical
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QED$_3$ definition

- QED$_3$ with with integer Chern-Simons level k:

$$\int d^3x \left[\frac{F_{\mu\nu}F^{\mu\nu}}{4e^2} - \frac{ik}{4\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + \mathcal{L}_{\text{matter}} \right] ,$$

  - $\mathcal{L}_{\text{scalar}} = \frac{\sigma^2}{4\lambda} + |(\nabla_\mu - iA_\mu)\phi^i|^2 + (\frac{1}{4} + i\sigma)|\phi^i|^2$, where N complex scalars ϕ_i and λ is real scalar Hubbard-stratonovich for ϕ^4 term.
  - $\mathcal{L}_{\text{fermion}} = -\bar{\psi}_i(i\not\nabla + \not A)\psi_i$ for even N 2-component complex ψ_i.

- $e, \lambda \to \infty$ when we flow to IR, bc F^2 and σ^2 are irrelevant.

- Can construct operators from ϕ_i, ψ_i, σ, and A_μ in irreps of $SU(N)$ flavor symmetry, compute correlators at large N using Feynman diagrams [Halperin, Lubetsky, Ma '74; Kaul, Sachdev '08] .

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  - All fields in Lagrangian uncharged under $U(1)_T$.
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  - Dirac quantization condition requires $q \in \mathbb{Z}/2$.
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Δ_q from $S^2 \times \mathbb{R}$

- The state-operator correspondence relates M_q on $\mathbb{R}^3$ to state on S^2 Hilbert space with $4\pi q$ magnetic flux, s.t. Δ_q given by energy on $S^2 \times \mathbb{R}$ with $4\pi q$ flux [Borokhov, Kapustin, Wu '02] .
- Chern-Simons term contributes $2qk$ to Gauss law constraint, so need to dress vacuum with matter to make gauge invariant.
  - Fermion zero modes contribute qN.
- Consider thermal free energy $F_q \equiv \frac{-\log Z}{\beta}$ on $S^2 \times S^1_\beta$ with $4\pi q$ flux, where $\beta \equiv 1/T$ is length of S^1 [SMC, Iliesiu, Mezei, Pufu '17] .
- After integrating out matter, can compute F_q from large N saddle point, s.t. holonomy of gauge field acts as chemical potential for matter fixed by saddle condition to cancel gauge charge.
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Leading order free energy

- From saddle we get energy (scaling dimension) and entropy:

$$F_q = NF_q^{(0)} + F_q^{(1)} + \ldots\,, \qquad F_q^{(0)} = \Delta_q^{(0)} - \frac{1}{\beta} S_q^{(0)} + O(e^{-\beta})\,.$$

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$$S^{(0)}_{\text{fermion}} = -d_{\tilde{j}} \left(\xi_{\tilde{j}} \log \xi_{\tilde{j}} + (1-\xi_{\tilde{j}})\log[1-\xi_{\tilde{j}}]\right) .$$

  - Δ is casimir energy plus matter dressing to cancel $2qk$ gauge flux (extra $-N$ for fermions due to zero modes).
  - Entropy is irreps from different ways to contract indices of dressing.

Subleading

- Subleading $F_q^{(1)}$ from fluctuations around saddle, numerically compute sum/integral to get scaling dimension:

$$\Delta_q^{(1)} = \int \frac{d\omega}{2\pi} \sum_{\ell=0}^{\infty} (2\ell+1) \log\det\left[\frac{\mathbf{K}_\ell^{q,\kappa}(\omega)}{\mathbf{K}_\ell^{0,\kappa}(\omega)}\right].$$

- For $k=0$, Δ computed to subleading order $O(N^0)$ for first fermions [Pufu '13] then scalars [Dyer, Mezei, Pufu, Sachdev '15].
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Comparison of Δ_q to duality: particle/vortex

- Scalar QED3 with $N = 1$ and $k = 0 \Leftrightarrow$ critical $O(2)$ Wilson Fisher.
- $M_q \Leftrightarrow$ lowest dimension operator made of $2q$ complex bosons ϕ:
  - $M_{1/2} \Leftrightarrow \phi$, and $M_1 \Leftrightarrow \phi\phi$, and $M_{3/2} \Leftrightarrow \phi\phi\phi$.
- All these operators are unique scalars, so no degeneracy breaking terms in monopole calculation.
- $O(2)$ operators computed for $q \leq 2$ at high precision from numerical bootstrap [SMC, Landry, Liu, Poland, DSD, Su, Vichi '20; Liu, Meltzer, Poland, DSD '20] .
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Evidence for particle/vortex from monopoles

q	$\Delta^{(0)}_{q,0}$	$\Delta^{(1)}_{q,0}$	$N = 1$	$O(2)$	Error (%)
$1/2$	0.12459	0.38147	0.50609	0.519130434	2.5
1	0.31110	0.87452	1.1856	1.23648971	4.1
$3/2$	0.54407	1.4646	2.0087	2.1086(3)	4.7
2	0.81579	2.1388	2.9546	3.11535(73)	5.2
$5/2$	1.1214	2.8879	4.0093	4.265(6)	5.8
3	1.4575	3.7053	5.1628	5.509(7)	6.3
$7/2$	1.8217	4.5857	6.4074	6.841(8)	6.3
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5	3.0638	7.5665	10.630	11.399(10)	6.7

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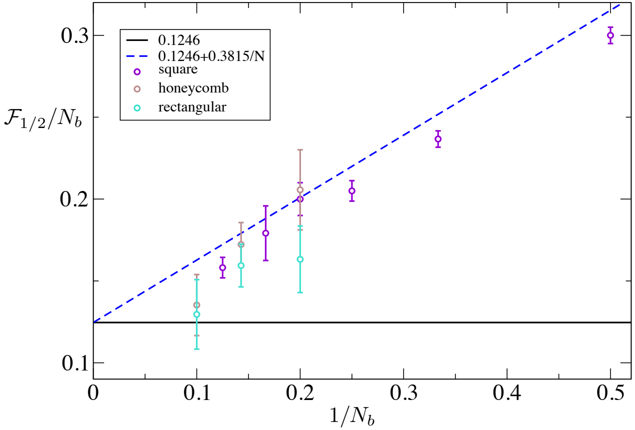
Comparison to lattice for $N > 1$ and $k = 0$



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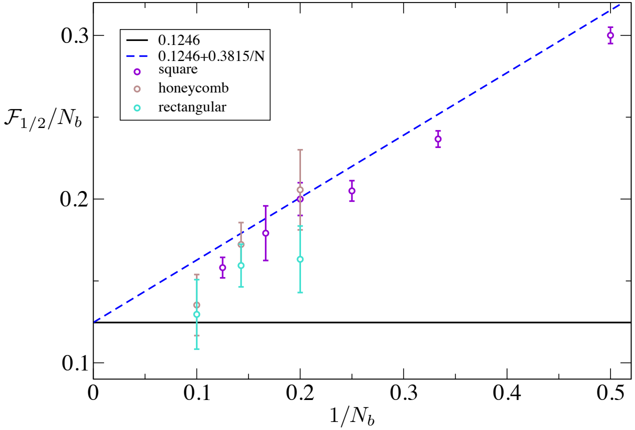

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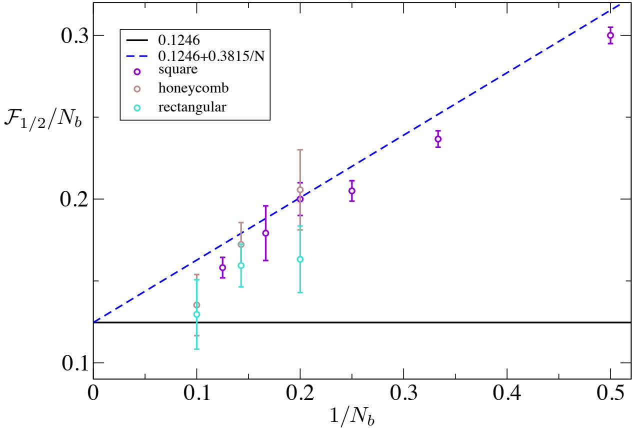

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- Scalar QED3 with $N = k = 1 \Leftrightarrow$ Free complex 2 component fermion ψ_α.
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- For higher q need to dress with derivatives bc of antisymmetry, so degenerate operators with same q and dimension, e.g. for $q = 2$:
  1. $\epsilon^{\alpha\beta}\psi_\alpha\psi_\beta\epsilon^{\gamma\delta}\partial_\mu\psi_\gamma\partial_\nu\psi_{\gamma\delta}$ has $\Delta = 6$ and spin 2.
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Operators in free fermion theory

- We can determine spectrum of free fermion theory by looking at free energy on $S^2 \times \mathbb{R}$ in presence of background $U(1)$ flux q.
- Fermionic modes of spin $j = 1/2, 3/2, \ldots$ have eigenvalue $\lambda_j = j + 1/2$, charge 1/2, and $2j + 1$ in each energy shell.
- Operators with charge q that correspond to states of n filled energy shells are unique scalars have charge and dimension:

$$q = \sum_{j=1/2}^{n-1/2} (2j+1) = n(n+1)/2\,, \qquad \text{e.g.} \quad q = 1, 3, 6, 10, \ldots$$

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- Fermionic modes of spin $j = 1/2, 3/2, \ldots$ have eigenvalue $\lambda_j = j + 1/2$, charge 1/2, and $2j + 1$ in each energy shell.
- Operators with charge q that correspond to states of n filled energy shells are unique scalars have charge and dimension:

$$q = \sum_{j=1/2}^{n-1/2} (2j+1) = n(n+1)/2\,, \qquad \text{e.g.} \quad q = 1, 3, 6, 10, \ldots$$

$$\Delta = \sum_{j=1/2}^{n-1/2} (2j+1)\lambda_j = \frac{2}{3} q\sqrt{1+8q}\,, \quad \text{e.g.} \quad \Delta = 2, 10, 28, 60, \ldots$$

- Operators that correspond to states of partially filled energy shells will have spin and degeneracy corresponding to valence modes.

Evidence for 3d bosonization from monopoles

q	$\Delta^{(0)}_{q,1}$	$\Delta^{(1)}_{q,1}$	$N=1$	Fermion	Error (%)
$1/2$	1	-0.2789	0.7211	1	28
1	2.5833	-0.6312	1.952	2	2.4
$3/2$	4.5873	-1.052	3.535	4	15
2	6.9380	-1.534	5.404	6	9.9
$5/2$	9.5904	-2.070	7.52	8	6.0
3	12.514	-2.655	9.859	10	1.4
6	34.727	-7.032	27.70	28	1.1
10	74.141	-14.71	59.43	60	0.95
15	135.67	-26.63	109.04	110	0.87
21	224.23	-43.75	180.5	182	0.82

- Purple are unique scalar operators (i.e. filled energy shells)
- Find match for unique scalars, that improves with q.

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Match for other q

- Operators in free fermion theory that NOT unique scalars do not match our monopole calculation (tho mismatch shrinks with q).
- This could be because of the degeneracy breaking term in the large N calculation, that we have not taken into account.
- If we take $\Delta_q^{\text{free}} = \frac{2}{3}q\sqrt{1+8q}$ of unique scalars in free fermion theory, which only valid for $q = 1, 3, 6, \ldots,$ and analytically continue to general q then we get precise match now for all q:

$$\Delta_{1/2}^{\text{ferm}} = .7454,\ \Delta_{3/2}^{\text{ferm}} = 3.606,\ \Delta_{2}^{\text{ferm}} = 5.498,$$
$$\Delta_{1/2}^{\text{mono}} = .7211,\ \Delta_{3/2}^{\text{mono}} = 3.535,\ \Delta_{2}^{\text{mono}} = 5.404,$$

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Application to Neel-VBS phase transition

- Standard Landau-Ginzburg phase transition has symmetry G preserved in one phase, broken in other, e.g. critical $O(N)$ model.
- Phase transitions described by QED_3 for $N > 1$ violates this, bc $SU(N)$ broken in one phase, but $U(1)_T$ broken in the other.
- For materials with this phase transition, gauge fields emergent at critical point, i.e. deconfined [Senthil, Vishwanath, Balents, Sachdev, Fisher '04]
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Our bootstrap approach

- Bootstrap correlators of $SO(5)$ singlet s, vector v, rank-2 t, which gives access also to rank-3 t_3 and rank-4 t_4.
- Assume one relevant s (so two relevant $SU(2) \times U(1)_T$ singlets), one relevant v, t, t_3, everything else irrelevant.
  - Assumptions motivated by large N estimates of Δ_q, which correspond to rank-$2q$ operators.
- 29 crossing equations, use `Skydive` [Liu, DSD, Su, Rees '23] to get allowed region in space of $\left\{\Delta_v, \Delta_s, \Delta_t, \Delta_{t_3}, \frac{\lambda_{sss}}{\lambda_{vvt}}, \frac{\lambda_{tts}}{\lambda_{vvt}}, \frac{\lambda_{vvs}}{\lambda_{vvt}}, \frac{\lambda_{ttt}}{\lambda_{vvt}}\right\}$.
- Fix $\Delta_v = .63$ from large N for $\Delta_{1/2}$, and maximize Δ_t to look at boundary of allowed region and read off $\Delta_s, \Delta_t, \Delta_{t_3}, \Delta_{t_4}$.
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Our bootstrap results

	Δ_v	Δ_t	Δ_{t_3}	Δ_{t_4}	Δ_s
Bootstrap ’23	0.630^*	1.519	2.598	3.884	2.359
Large N	0.630	1.497	2.552	3.770	–
Bootstrap ’24	0.595^*	1.409	2.388	3.543	2.179
Lattice	0.607(4)	1.417(7)	–	3.723(11)	2.273(4)
Fuzzy Sphere	0.584	1.454	2.565	3.885	2.845

- We input one value Δ_v, to get predictions for three values Δ_t, Δ_{t_3}, Δ_{t_4} that all match large N! Plus prediction for relevant Δ_s.
- Our prediction verified by lattice study [Sandvik et al ’24] !
    - We include new bootstrap results using smaller Δ_v for comparison.
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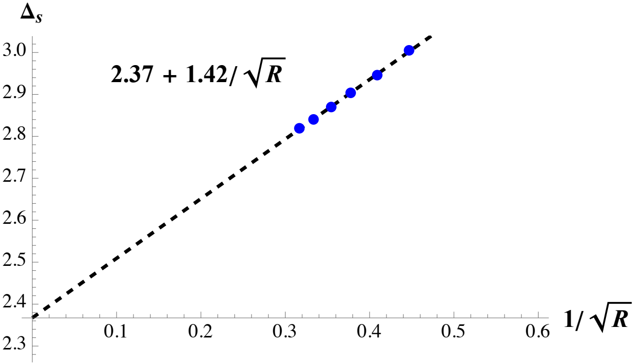

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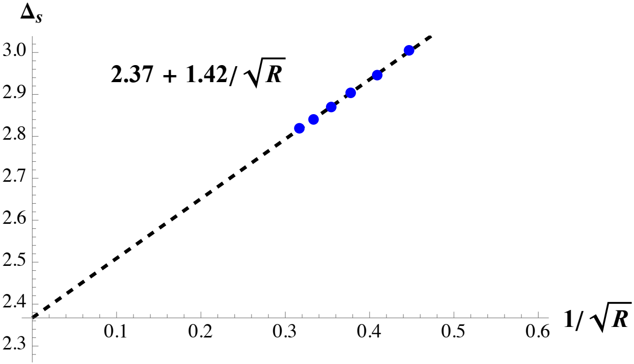

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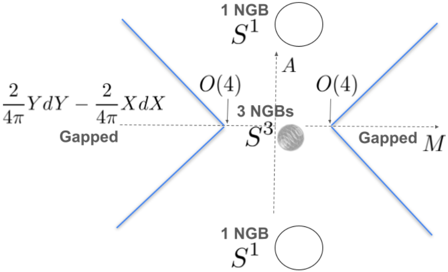

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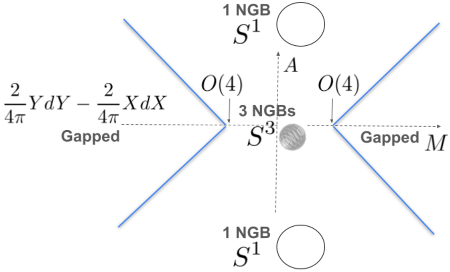

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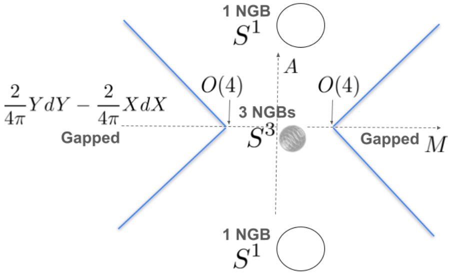

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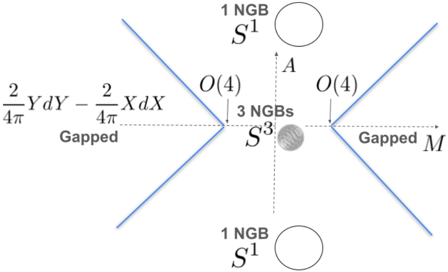

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Evidence from monopoles

q	$2\Delta^{(0)}_{q,0}$	$\Delta^{(1)}_{q,0}$	$N=2$	$O(4)$	Error (%)
$1/2$	0.5302	−0.038138	0.492062	0.515(3)	4.5
1	1.3463	−0.19340(3)	1.1529	1.185(4)	2.7
$3/2$	2.37286	−0.42109(4)	1.95177	1.989(5)	1.9
2	3.5738	−0.70482(9)	2.86898	2.915(6)	1.6
$5/2$	4.9269	−1.0358(2)	3.8911	3.945(6)	1.4
3	6.41674	−1.4082(2)	5.00854	5.069(7)	1.2
$7/2$	8.03182	−1.8181(2)	6.21372	6.284(8)	1.1
4	9.76308	−2.2623(3)	7.50078	7.575(9)	1.0
$9/2$	11.6032	−2.7384(3)	8.86482	8.949(10)	0.9
5	13.5462	−3.2445(3)	10.3017	10.386(11)	0.8

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Evidence from monopoles

q	$2\Delta^{(0)}_{q,0}$	$\Delta^{(1)}_{q,0}$	$N = 2$	$O(4)$	Error (%)
$1/2$	0.5302	-0.038138	0.492062	0.515(3)	4.5
1	1.3463	$-0.19340(3)$	1.1529	1.185(4)	2.7
$3/2$	2.37286	$-0.42109(4)$	1.95177	1.989(5)	1.9
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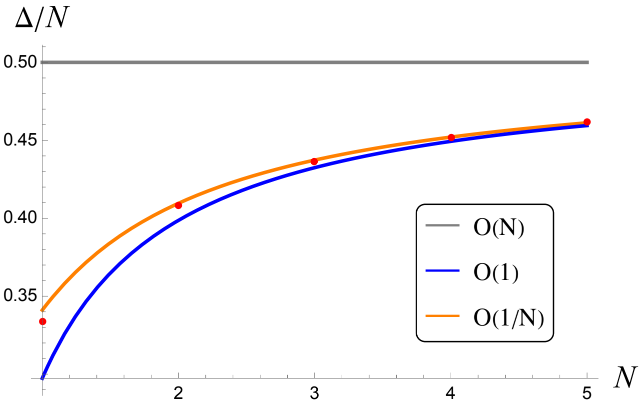

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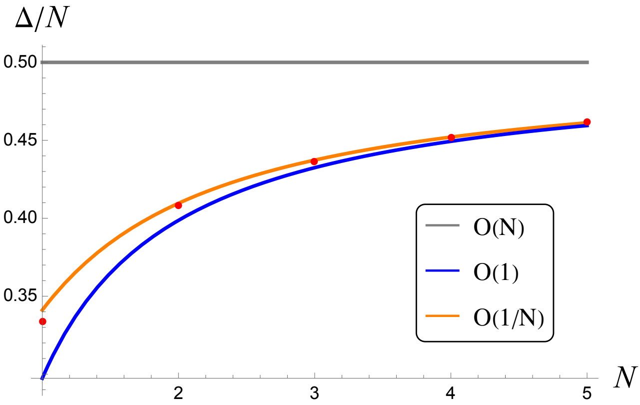

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