Three-point Functions in ABJM Theory and Integrable Boundary States

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Based on **JW** and Peihe Yang, [arxiv: 2408.03643]

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- The conformal dimensions of the operators (determining the two-point functions) and structure constants (OPE coefficients) are called conformal data.
- At weak coupling, they can be computed using perturbation theory.
- For theories with holographic duals, the strong coupling limit of conformal data can be calculated using weakly coupling gravity or string theory.

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- It is also needed when the coupling constants are around order one.

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- For $\mathcal{N}=4$ super Yang-Mills and ABJM theories, integrability is a very powerful non-perturbative tools (mainly in the planar limit).
- This also applies to some cousins (orbifolds, β/γ-deformations, fishnet theories...) of the $\mathcal{N}=4$ SYM and ABJM theories.

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- In the string theory side, integrability means that the worldsheet theory of type IIB superstring on $AdS_5 \times S^5$ in the free limit is a two-dimensional integrable field theory. [Benna, Polchinski and Roiban, 03]

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- The case for ABJM theory is in the same spirit but much more complicated and hard. [Minahan, Zarembo, 08][Bak, Rey, 08][Gromov, Vieira, 08], $\cdots$

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- However the three-point functions of single-trace operators in ABJM theory from integrability were reraly studied. Before our work, only the correlators in the $SU(2)\times SU(2)$ sector were studied [Bissi, Kristjansen, Martirosyan, Orselli, 12].

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- In this talk, we will study a class of three-point functions in ABJM theory from the viewpoint of integrable boundary states.

Integrable boundary state (IBS)

- Integrable boundary states [Piroli, Pozsgay, Vernier, 17] of spin chain play an important role in both quantum quench dynamics and AdS/CFT correspondence. (Integrable boundary states in field theory were first studied in [Ghoshal, Zamolodchikov 93].)

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- The state after $t = 0$ is

$$|\Psi(t)\rangle = \exp(-\mathrm{i}\hat{H}t)|\Psi\rangle = \sum_{\alpha} \exp(-\mathrm{i}E_\alpha t)\langle\psi_\alpha|\Psi\rangle|\psi_\alpha\rangle \tag{1}$$

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  where $|\psi_\alpha\rangle$ is the normalized eigen-state of H with eigen-value E_α (the case with degeneracy can be treated similarly).
- If H is integrable, generically $|\psi_\alpha\rangle$ can be parameterized by Bethe roots, $\mathbf{u_1}, \cdots, \mathbf{u_r}$, and E_α is a function of these roots.

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- In $\mathcal{N} = 4$ SYM and ABJM theory many correlation functions can be also expressed as the overlap between a boundary state $\langle \Psi |$ and a Bethe state $|\mathbf{u_1}, \cdots, \mathbf{u_r}\rangle$.

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- In $\mathcal{N} = 4$ SYM and ABJM theory many correlation functions can be also expressed as the overlap between a boundary state $\langle \Psi |$ and a Bethe state $|\mathbf{u_1}, \cdots, \mathbf{u_r}\rangle$.
- If this boundary state $\langle \Psi |$ is integrable, we may have compact formulas for such correlation functions.

IBS in AdS/CFT

theories	domain wall one-point functions	other disordered operator one-point functions
$\mathcal{N} = 4$ SYM	[de Leeuw, Kristjansen, Zarembo, 15], $\cdots$	't Hooft loops: [Kristjansen, Zarembo, 23] surface operators: unexplored
ABJM	[Kristjansen, Vu, Zarembo, 21]	vortex loops: unexplored

IBS in AdS/CFT

theories	1-pt functions on the Coulumb branch	$\langle W[C]O\rangle$
$\mathcal{N}=4$ SYM	[Ivanovskiy, et al., 24]	[Jiang, Komatsu, Vescovi, to appear]
ABJM	unexplored	[Jiang, **JW**, Yang, 23]

$W[C]$ is a certain BPS Wilson loop, and $\mathcal{O}$ is a generic non-BPS single-trace operator.

IBS in AdS/CFT

theories	$\langle \mathcal{D}^\circ \mathcal{D}^\circ \mathcal{O} \rangle$	$\langle \mathcal{O}^\circ \mathcal{O}^\circ \mathcal{O} \rangle$
$\mathcal{N}=4$ SYM	[Jiang, Komatsu, Vescovi, 19]	unexplored
ABJM	[Yang, Jiang, Komatsu, **JW**, 21]	[**JW**, Yang, **this talk**]

$\mathcal{D}^\circ$'s and $\mathcal{O}^\circ$'s are BPS determinant operators (dual to giant gravitons) and BPS single-trace operators, respectively.

ABJM theory

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- The matter fields include complex scalars Y^I and spinors ψ_I ($I = 1, \cdots, 4$) in the bi-fundamental representation of the gauge group.
- ABJM gave strong evidence to support this theory to be the low energy effective theory of N M2-branes putting at the tip of $\mathbf{C}^4/\mathbf{Z}_k$.

Quiver diagram of ABJM theory

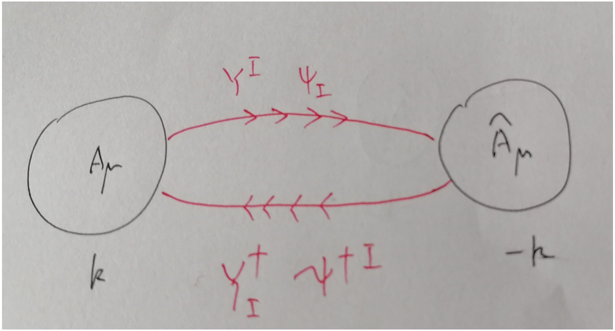

Figure: The quiver diagram of ABJM theory.

Single-trace operators

- The single-trace operator in the scalar sector of the ABJM theory is
$$\mathcal{O}_C = C^{J_1 \cdots J_L}_{I_1 \cdots I_L} \mathrm{Tr}(Y^{I_1} Y^\dagger_{J_1} \cdots Y^{I_L} Y^\dagger_{J_L}) \,. \qquad (2)$$

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- The cyclicity property of the trace can be used to choose C to be invariant under the following simultaneous cyclic shift of the upper and lower indices, $C^{J_1 \cdots J_L}_{I_1 \cdots I_L} = C^{J_2 \cdots J_L J_1}_{I_2 \cdots I_L I_1}$.

Chiral Primary Operators

- When the tensor C is invariant under the respective permutations among the upper and the lower indices, and traceless,

$$C^{J_1\cdots J_L}_{I_1\cdots I_L} = C^{J_1\cdots J_L}_{(I_1\cdots I_L)} = C^{(J_1\cdots J_L)}_{I_1\cdots I_L}\,,\; C^{J_1\cdots J_L}_{I_1\cdots I_L}\,\delta^{I_1}_{J_1} = 0\,, \tag{3}$$

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- A natural choice of such symmetric traceless tensor C is in terms of polarization vectors n_I and $\bar{n}^I$,

$$C^{J_1 \cdots J_L}_{I_1 \cdots I_L} = n_{I_1} \cdots n_{I_L} \bar{n}^{J_1} \cdots \bar{n}^{J_L}\,, \tag{4}$$

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  with BPS condition $n\cdot\bar{n} = 0$.
- Notice that $\bar{n}$ does not need to be the complex conjugation of n.

Two point functions

- With this choice, the BPS operator becomes

$$\mathcal{O}_L^\circ(x,n,\bar{n}) = \operatorname{tr}\left(\left(n\cdot Y\bar{n}\cdot Y^\dagger\right)^L\right) . \tag{5}$$

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- The two-point function of $\mathcal{O}_L^\circ$'s is constrained by symmetries to take the form,

$$\langle \mathcal{O}_{L_1}^\circ(x_1)\mathcal{O}_{L_2}^\circ(x_2)\rangle = \delta_{L_1,L_2}\mathcal{N}_{\mathcal{O}_{L_1}^\circ}(d_{12}d_{21})^{L_1}, \tag{6}$$

with the definitions

$$d_{ij} = \frac{n_i\cdot\bar{n}_j}{|x_{ij}|}\,,\; x_{ij} = x_i - x_j\,. \tag{7}$$

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$$\mathcal{O}^\circ_L(x,n,\bar{n}) = \operatorname{tr}\left(\left(n\cdot Y \bar{n}\cdot Y^\dagger\right)^L\right) . \tag{5}$$

- The two-point function of $\mathcal{O}^\circ_L$'s is constrained by symmetries to take the form,

$$\langle \mathcal{O}^\circ_{L_1}(x_1)\mathcal{O}^\circ_{L_2}(x_2)\rangle = \delta_{L_1,L_2}\mathcal{N}_{\mathcal{O}^\circ_{L_1}}(d_{12}d_{21})^{L_1}, \tag{6}$$

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- At tree-level in the planar limit, we have

$$\mathcal{N}_{\mathcal{O}^\circ_L} = L\lambda^{2L}\,. \tag{8}$$

Non-BPS Operators

- We consider a non-BPS operator

$$\mathcal{O}_C = C^{J_1\cdots J_L}_{I_1\cdots I_L}\mathrm{Tr}(Y^{I_1}Y^\dagger_{J_1}\cdots Y^{I_L}Y^\dagger_{J_L})\,. \tag{9}$$

which can be mapped to the following state

$$|\mathcal{O}_C\rangle = C^{J_1J_2\cdots J_L}_{I_1I_2\cdots I_L}|I_1,\bar{J}_1,\cdots,I_L,\bar{J}_L\rangle\,, \tag{10}$$

of the $SU(4)$ alternating spin chain. The Hamiltonian on this spin chain is from the planar two-loop dilatation operator in the scalar sector. This Hamiltonian has been proven to be integrable [Minahan, Zarembo, 08][Bak, Rey, 08]. And the above state is taken as one of the eigen-states of this Hamiltonian.

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- Generically this state can be parametrized by the solution $\mathbf{u}, \mathbf{w}, \mathbf{v}$ to the Bethe ansatz equations and zero-momentum condition,

$$|\mathcal{O}\rangle = |\mathbf{u}, \mathbf{w}, \mathbf{v}\rangle \,. \tag{11}$$

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- We put all operators along the line $x^\mu = (0, 0, a)$.
- When we translate an operator from the origin to the point $(0, 0, a)$, we perform the following transformation,

$$Y^1 \to Y^1 + \kappa a Y^4, \; Y_4^\dagger \to Y_4^\dagger - \kappa a Y_1^\dagger. \tag{12}$$

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- From now on, we choose $\kappa = 1$. The κ-depedenece can be recovered by dimensional analysis.

Three-point functions

- Based on the conformal symmetry and R-symmetry, the normalized correlation function of three generic single-trace operators in the twisted-translated frame should take the following form,

$$\frac{\langle\hat{\mathcal{O}}_1(a_1)\hat{\mathcal{O}}_2(a_2)\hat{\mathcal{O}}_3(a_3)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}_1}\mathcal{N}_{\mathcal{O}_2}\mathcal{N}_{\mathcal{O}_3}}}=\frac{\mathcal{C}_{123}}{a_{12}^{\gamma_{12|3}}a_{23}^{\gamma_{23|1}}a_{31}^{\gamma_{31|2}}}\,, \tag{13}$$

  where

$$\gamma_{ij|k}:=(\Delta_i+\Delta_j-\Delta_k)-(J_i+J_j-J_k)\,, \tag{14}$$

  and J is a $U(1)$ R-charge which assigns charges $(1/2,0,0,-1/2)$ to $Y^1,\cdots,Y^4$. [Kazama, Komatsu, Nishimura, 14][Yang, Jiang, Komatsu, **JW**, 21]

Three-point functions

- The main focus of this talk is on three-point functions of two $1/3$-BPS single-trace operators $\hat{\mathcal{O}}^{\circ}_{L_i}, i = 1, 2$ and one non-BPS operator $\hat{\mathcal{O}}_3$.
- For this special case we have

$$\frac{\langle \hat{\mathcal{O}}^{\circ}_1(a_1)\hat{\mathcal{O}}^{\circ}_2(a_2)\hat{\mathcal{O}}_3(a_3)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}^{\circ}_1}\mathcal{N}_{\mathcal{O}^{\circ}_2}\mathcal{N}_{\mathcal{O}_3}}} = \mathcal{C}_{123}\left(\frac{a_{12}}{a_{23}a_{31}}\right)^{\Delta_3 - J_3} . \tag{15}$$

Three point functions

- In the large N limit, the tree-level three-point function

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- Notice that $l_{12|3}, l_{23|1}, l_{31|2}$ always have the same odevity. This behavior contrasts with that in $\mathcal{N} = 4$ SYM theory.

Wick contractions

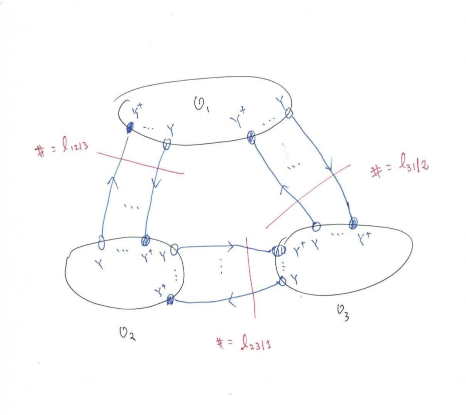

Figure: Tree-level planar Wick contractions.

Boundary states

- The three point function $\langle \hat{\mathcal{O}}_1^\circ(a_1)\hat{\mathcal{O}}_2^\circ(a_2)\hat{\mathcal{O}}_3(a_3)\rangle$ can be expressed using the overlap between a boundary state and a Bethe state. When $l_{ij|k}$'s are even and $2 \leq l_{31|2} \leq 2L_3 - 2$, we have

$$\langle \hat{\mathcal{O}}_1^\circ(a_1)\hat{\mathcal{O}}_2^\circ(a_2)\hat{\mathcal{O}}_3(a_3)\rangle = \frac{(-1)^{\frac{l_{12|3}}{2}} L_1 L_2 \lambda^{\sum_{i=1}^3 L_i}}{N|a_1|^{l_{31|2}}|a_2|^{l_{23|1}}} \langle \mathcal{B}^{\text{even}}_{l_{31|2}}|\mathbf{u},\mathbf{w},\mathbf{v}\rangle\,. \tag{17}$$

Boundary states

- To make the structure of $|\mathcal{B}^{\text{even}}_{l_{31|2}}\rangle$ clear, We first define

$$
\begin{aligned}
&\langle \bar{n}_1 @\{1,2,\cdots,m\}, n_1 @\{1,2,\cdots,m\}| = \\
&(\bar{n}_1)^{I_1}(n_1)_{J_1}\cdots(\bar{n}_1)^{I_m}(n_1)_{J_m}(\bar{n}_2)^{I_{m+1}}(n_2)_{J_{m+1}} \\
&\cdots(\bar{n}_2)^{I_1}(n_2)_{J_L}\langle I_1, J_1, \cdots, I_L, J_L|\,.
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- And

$$\begin{aligned}&U_{\text{even}}|I_1,J_1,I_2,J_2\cdots,I_{L-1},J_{L-1},I_L,J_L\rangle \\ &=|I_1,J_2,I_2,J_3,\cdots,I_{L-1},J_L,I_L,J_1\rangle\,,\end{aligned} \tag{19}$$

$$\begin{aligned}&U_{\text{odd}}|I_1,J_1,I_2,J_2\cdots,I_{L-1},J_{L-1},I_L,J_L\rangle \\ &=|I_2,J_1,I_3,J_2,\cdots,I_L,J_{L-1},I_1,J_L\rangle\,,\end{aligned} \tag{20}$$

where U_{even} has already been introduced in [Jiang, **JW**, Yang, 23].

Boundary states

- Then the boundary state $|\mathcal{B}^{\text{even}}_{l_{31|2}}\rangle$ in the considered case is,

$$\langle\mathcal{B}^{\text{even}}_{l_{23|1}}| = \langle\mathcal{B}^{\text{even},\,a}_{l_{23|1}}| + \langle\mathcal{B}^{\text{even},\,b}_{l_{23|1}}| \tag{21}$$

with

$$\langle\mathcal{B}^{\text{even},\,a}_{l}| = \sum_{j=0}^{L-1}\langle\bar{n}_1@\{1,2,\cdots,l/2\}, n_1@\{1,2,\cdots,l/2\}|(U_{\text{even}}U_{\text{odd}})^j\,, \tag{22}$$

$$\langle\mathcal{B}^{\text{even},\,b}_{l}| = \langle\mathcal{B}^{\text{even},\,a}_{l}|U_{\text{even}}\,. \tag{23}$$

Boundary states

- When $l_{31|2} = 0$, the boundary state is

$$\langle \mathcal{B}_0^{\text{even}} | = L(\bar{n}_2)^{I_1}(n_2)_{J_1} \cdots (\bar{n}_2)^{I_L}(n_2)_{J_L} \langle I_1, J_1, \cdots, I_L, J_L | . \quad (24)$$

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- Then boundary state for the case $l_{31|2} = 2L_3$ is

$$\langle \mathcal{B}_{2L_3}^{\text{even}} | = L \langle n_1 @ \{1, 2, \cdots, L_3\}, \bar{n}_1 @ \{1, 2, \cdots, L_3\} | \, . \tag{25}$$

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- We temporarily ignore this mixing.

Boundary states

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- For example, when $l_{31|2} = 1$, we have

$$\langle \mathcal{B}_1^{\text{odd}} | = \langle \mathcal{B}_1^{\text{odd},\,a} | - \langle \mathcal{B}_1^{\text{odd},\,b} | \,, \tag{26}$$

$$\langle \mathcal{B}_1^{\text{odd},\,a} | = \sum_{l=1}^{L} \langle \bar{n}_1 @ l | \,, \tag{27}$$

$$\langle \mathcal{B}_1^{\text{odd},\,b} | = \sum_{l=1}^{L} \langle n_1 @ l |, \tag{28}$$

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- We have numerical results to support that the boundary states the other cases are not integrable.

Special extremal correlators

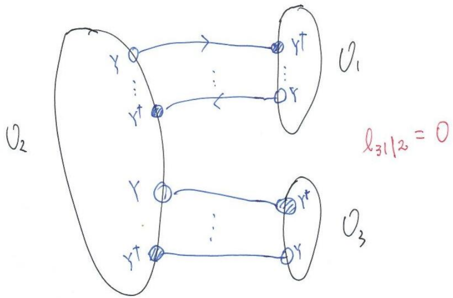

Figure: Special extremal correlators.

Special next-to-extremal correlators

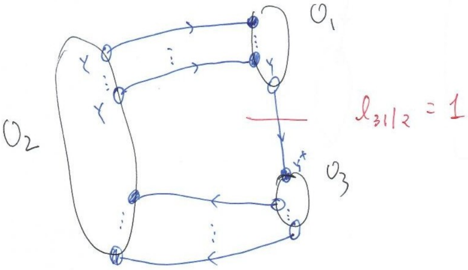

Figure: Special next-to-extremal correlators.

- The planar two-loop dilatation operators in the scalar sector of ABJM theory can be described by the $SU(4)$ alternating spin chain with the integrable Hamiltonian,

$$H = \frac{\lambda^2}{2} \sum_{l=1}^{2L} \left(2 - 2\mathbb{P}_{l,l+2} + \mathbb{P}_{l,l+2}\mathbb{K}_{l,l+1} + \mathbb{K}_{l,l+1}\mathbb{P}_{l,l+2}\right). \qquad (30)$$

ABJM spin chain

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- Gauge invariance requires that single-trace operators correspond to an alternating spin chain where odd and even sites are in the fundamental and anti-fundamental representations of the $SU(4)$ R-symmetry group, respectively.

R-matrices

- The $SU(4)$ alternating chain has four R matrices,

$$
\begin{aligned}
R_{0j}(\lambda) &= \lambda\mathbb{I} + \mathbb{P}_{0j}\ , \\
R_{0\bar{j}}(\lambda) &= -(\lambda+2)\mathbb{I} + \mathbb{K}_{0\bar{j}}\ . \\
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- Here, 0 and $\bar{0}$ denote the auxiliary space in the fundamental and anti-fundamental representations of the $SU(4)$, respectively.

Monodromy matrices and transfer matrices

- We define two monodromy matrices as

$$T_0(\lambda) = R_{01}(\lambda) R_{0\bar{1}}(\lambda) \cdots R_{0L}(\lambda) R_{0\bar{L}}(\lambda) \quad (32)$$

$$\bar{T}_{\bar{0}}(\lambda) = R_{\bar{0}1}(\lambda) R_{\bar{0}\bar{1}}(\lambda) \cdots R_{\bar{0}L}(\lambda) R_{\bar{0}\bar{L}}(\lambda). \quad (33)$$

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- We have

$$[\tau(u), \tau(v)] = [\bar{\tau}(u), \bar{\tau}(v)] = [\tau(u), \bar{\tau}(v)] = 0\,, \tag{35}$$

and the previous Hamiltonian can be obtained from the series expansion of $\log(\tau(u)\bar{\tau}(u))$ at $u = 0$.

Decomposition of $\tau(\lambda)$

- Let us decompose $\tau(\lambda)$ as

$$\tau(\lambda) = \sum_{m=0}^{L}\sum_{n=0}^{L} \lambda^{L-m}(-\lambda-2)^{L-n}\mathcal{O}_{m,n}\,. \tag{36}$$

  Here for each term of $\mathcal{O}_{m,n}$, there are m $\mathbb{P}$'s and n $\mathbb{K}$'s inside the trace.

- We have

$$\begin{aligned}\tau(-\lambda-2) &= \sum_{m=0}^{L}\sum_{n=0}^{L}(-\lambda-2)^{L-m}\lambda^{L-n}\mathcal{O}_{m,n} \\ &= \sum_{m=0}^{L}\sum_{n=0}^{L}\lambda^{L-m}(-\lambda-2)^{L-n}\mathcal{O}_{n,m}\,.\end{aligned}$$

Integrable boundary states

- If $|\mathcal{B}\rangle$ satisfies the conditions,

$$\mathcal{O}_{m,n}|\mathcal{B}\rangle = \mathcal{O}_{n,m}|\mathcal{B}\rangle \,, \tag{37}$$

for any $0 \leq m, n \leq L$, then it satisfies The twisted integrable condition

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[Yang, 2022]

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[Yang, 2022]

- We ([Yang, **JW**, 2024]) proved that both $|\mathcal{B}_1^{\mathrm{odd},\,b}\rangle$ and $|\mathcal{B}_1^{\mathrm{odd},\,a}\rangle$ satisfy the conditions (37). So they are integrable, as well as $|\mathcal{B}_1^{\mathrm{odd}}\rangle$. This is one of our main results.

Integrable boundary states

- In the proof, we do not use the BPS conditions, $n_1 \cdot \bar{n}_1 = n_2 \cdot \bar{n}_2 = 0$, letting along the twisted-translated frame.

Integrable boundary states

- In the proof, we do not use the BPS conditions, $n_1 \cdot \bar{n}_1 = n_2 \cdot \bar{n}_2 = 0$, letting along the twisted-translated frame.
- So this proof show that $|\mathcal{B}_l^{\mathrm{odd},\,a}\rangle$ and $|\mathcal{B}_l^{\mathrm{odd},\,b}\rangle$ are integrable for $l = 1$ or $l = 2L_3 - 1$ without any constraints on n_i's and $\bar{n}_i$'s.

The overlaps

- The twisted-translated frame leads to the selection rule,

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- The following result from symmetry

$$\frac{\langle \hat{\mathcal{O}}_1^{\circ}(a_1)\hat{\mathcal{O}}_2^{\circ}(a_2)\hat{\mathcal{O}}_3(a_3)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}_1^{\circ}}\mathcal{N}_{\mathcal{O}_2^{\circ}}\mathcal{N}_{\mathcal{O}_3}}} = \mathcal{C}_{123}\left(\frac{a_{12}}{a_{23}a_{31}}\right)^{\Delta_3 - J_3} \,, \tag{40}$$

further leads to the constraints,

$$N_{\mathbf{u}} \leq \min(l_{31|2}, l_{23|1}) \,. \tag{41}$$

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further leads to the constraints,

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- Using this constraint and coordinate Bethe ansatz (CBA), all overlaps from the three point functions in the integrable cases are computed.

An example

- Consider the case with $l_{31|2} = N_{\mathbf{u}} = N_{\mathbf{w}} = N_{\mathbf{v}} = 1$, the Bethe ansatz equations are

$$1 = \left(\frac{u+\frac{\mathrm{i}}{2}}{u-\frac{\mathrm{i}}{2}}\right)^{L_3} \frac{u-w+\frac{\mathrm{i}}{2}}{u-w-\frac{\mathrm{i}}{2}}, \tag{42}$$

$$1 = \frac{w-u+\frac{\mathrm{i}}{2}}{w-u-\frac{\mathrm{i}}{2}} \tag{43}$$

$$1 = \left(\frac{v+\frac{\mathrm{i}}{2}}{v-\frac{\mathrm{i}}{2}}\right)^{L_3} \frac{v-w+\frac{\mathrm{i}}{2}}{v-w-\frac{\mathrm{i}}{2}}, \tag{44}$$

and the zero-momentum condition is

$$1 = \frac{u+\frac{\mathrm{i}}{2}}{u-\frac{\mathrm{i}}{2}} \frac{v+\frac{\mathrm{i}}{2}}{v-\frac{\mathrm{i}}{2}}. \tag{45}$$

An example

- The solutions are [Bak, Rey, 08]

$$u = -v = \frac{1}{2} \cot \frac{k\pi}{L_3 + 1}, \; w = 0 \,, \tag{46}$$

with $k = 1, \cdots, L_3$.

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with $k = 1, \cdots, L_3$.

- By constructing eigenstates via CBA and Gaudin formula for the norms [Yang, Jiang, Komatsu, **JW**, 21], we can get

$$\mathcal{C}_{123} = \frac{(-1)^{L_2+1}\mathrm{sgn}(a_1 a_2 a_{12})\sqrt{2L_1L_2L_3}\,\exp\frac{2\pi k\mathrm{i}}{L_3+1}}{N\sqrt{L_3+1}}\,. \tag{47}$$

Conclusion

- We found that the boundary state from the two BPS operators is intergrable only when the correlator is special extremal ($l_{31|2} = 0, 2L_3$) or special next-to-extremal ($l_{31|2} = 1, 2L_3 - 1$).

Conclusion

- We found that the boundary state from the two BPS operators is intergrable only when the correlator is special extremal ($l_{31|2} = 0, 2L_3$) or special next-to-extremal ($l_{31|2} = 1, 2L_3 - 1$).
- For these integrable case, we computed the three-point functions using the constraints from symmetries and CBA.

Outlook

- It should be interesting to revisit the three-point function $\langle \mathcal{O}^\circ \mathcal{O}^\circ \mathcal{O} \rangle$ in $\mathcal{N} = 4$ SYM [Escobedo, Gromov, Sever, Vieira, 10] to determine when the boundary state from the two BPS operators is integrable.

Outlook

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- It is also desirable to compute more general three-point functions in ABJM theory to aid the development of hexagon program [Basso, komatsu, Vieira, 15] for this theory.

Thanks for Your Attention !