Holographic correlators beyond (half-)maximal supersymmetry

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Based on WIP with Nikolay Bobev

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A 5d supergravity truncation: the 10-scalar model

[Bobev,Elvang,Kol,Olson,Pufu'16]

Full non-linear Lagrangian:

$$\mathcal{L} = -\frac{1}{4}R + 3(\partial\beta_1)^2 + (\partial\beta_2)^2 + \frac{1}{2}\mathcal{K}_{a\bar{b}}\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} - \mathcal{P}$$

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Kinetic terms encoded in Kähler metric

$$\mathcal{K}_{a\bar{b}} \equiv \frac{\partial^2 \mathcal{K}}{\partial z^a \partial \bar{z}^{\bar{b}}} \qquad \mathcal{K} = -\sum_{a=1}^{4} \log(1 - z^a \bar{z}^a)$$

$$z^1 = \tanh\left[\frac{1}{2}(\alpha_1 + \alpha_2 + \alpha_3 + \varphi - i\phi_1 - i\phi_2 - i\phi_3 + i\phi_4)\right]$$

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Scalar potential

$$\mathcal{P} = \frac{1}{8}e^{\mathcal{K}}\left[\frac{1}{6}\partial_{\beta_1}\mathcal{W}\partial_{\beta_1}\overline{\mathcal{W}} + \frac{1}{2}\partial_{\beta_2}\mathcal{W}\partial_{\beta_2}\overline{\mathcal{W}} + \mathcal{K}^{\bar{b}a}\nabla_a\mathcal{W}\nabla_{\bar{b}}\overline{\mathcal{W}} - \frac{8}{3}\mathcal{W}\overline{\mathcal{W}}\right]$$

written in terms of 'superpotential':

$$\begin{aligned}\mathcal{W} \equiv\ & \frac{1}{L}e^{2\beta_1+2\beta_2}\left(1 + z_1z_2 + z_1z_3 + z_1z_4 + z_2z_3 + z_2z_4 + z_3z_4 + z_1z_2z_3z_4\right)\\ &+ \frac{1}{L}e^{2\beta_1-2\beta_2}\left(1 - z_1z_2 + z_1z_3 - z_1z_4 - z_2z_3 + z_2z_4 - z_3z_4 + z_1z_2z_3z_4\right)\\ &+ \frac{1}{L}e^{-4\beta_1}\left(1 + z_1z_2 - z_1z_3 - z_1z_4 - z_2z_3 - z_2z_4 + z_3z_4 + z_1z_2z_3z_4\right)\end{aligned}$$

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$$\text{Step 1:}\quad \phi_1 \mapsto -\frac{\pi}{6} - \phi_1\,, \quad \beta_1 \mapsto \frac{\log(2)}{12} - \beta_1\,, \quad \beta_2 \mapsto \frac{\log(2)}{4} - \beta_2\,,$$

$$\text{Step 2:}\quad \beta_1 \mapsto \frac{1}{\sqrt{6}}\beta_1\,, \quad \beta_2 \mapsto \frac{1}{\sqrt{2}}\beta_2\,, \quad \Phi_i \mapsto \frac{\sqrt{3}}{2}\Phi_i\,,$$

$$\text{Step 3:}\quad \widehat{\beta} \equiv \frac{\beta_2 - \sqrt{3}\beta_1}{2}\,,$$

$$\rho_1 \equiv \frac{1}{2\sqrt{14}}\Big(\sqrt{7+\sqrt{7}}\,\beta_1 + \sqrt{3(7+\sqrt{7})}\,\beta_2 + 2\sqrt{7-\sqrt{7}}\,\phi_1\Big),$$

$$\rho_2 \equiv -\frac{1}{2\sqrt{14}}\Big(\sqrt{7-\sqrt{7}}\,\beta_1 + \sqrt{3(7-\sqrt{7})}\,\beta_2 - 2\sqrt{7+\sqrt{7}}\,\phi_1\Big).$$

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→ **resulting quadratic terms in the potential:**

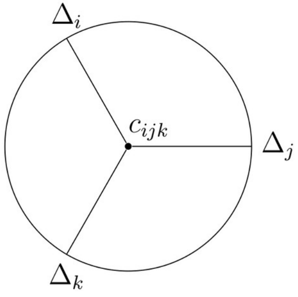
$$\mathcal{P}^{(2)} = \frac{1}{2}\Big[(4-2\sqrt{7})\rho_1^2 + 3\alpha_1^2 + (4+2\sqrt{7})\rho_2^2 - 4\widehat{\beta}^2 - \frac{15}{4}(\alpha_2^2+\alpha_3^2+\phi_2^2+\phi_3^2) - 3\phi_4^2\Big]$$

Spectrum around the IR solution

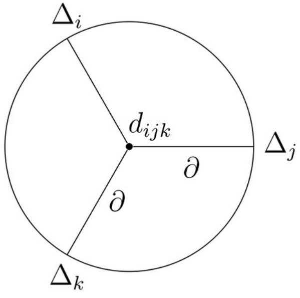
	$m^2 L^2_{\mathrm{LS}}$	Δ	$\mathcal{N}=1$ multiplet	$SU(2)_F\ U(1)_R$
α_2, α_3	$-\frac{15}{4}$	$\frac{3}{2}$	$L\bar{B}_1[\frac{3}{2};0,0;1]\otimes[1]^{(1)}+\text{c.c.}$	$\mathbf{3}_1\oplus\mathbf{3}_{-1}$
ϕ_2, ϕ_3	$-\frac{15}{4}$	$\frac{5}{2}$	(chiral)	$\mathbf{3}_{-1}\oplus\mathbf{3}_1$
$\widehat{\beta}$	-4	2	$A\bar{A}[2;0,0;0]\otimes[1]^{(0)}$	$\mathbf{3}_0$
			(flavour current)	
ϕ_4	-3	3	$L\bar{B}_1[3;0,0;2]\otimes[0]^{(2)}+\text{c.c.}$	$\mathbf{1}_2\oplus\mathbf{1}_{-2}$
φ	0	4	(chiral)	$\mathbf{1}_0\oplus\mathbf{1}_0$
ρ_1	$4-2\sqrt{7}$	$1+\sqrt{7}$	$L\bar{L}[\widehat{\Delta};0,0;0]\otimes[0]^{(0)}$	$\mathbf{1}_0$
α_1	3	$2+\sqrt{7}$	(long)	$\mathbf{1}_2\oplus\mathbf{1}_{-2}$
ρ_2	$4+2\sqrt{7}$	$3+\sqrt{7}$		$\mathbf{1}_0$

Holographic 3pt-functions: the bulk computation

Evaluation of 3pt-Witten diagrams yields: [Freedman,Mathur,Matusis,Rastelli'98]



$$A_1 = -\frac{1}{\sqrt{\eta}} \cdot \frac{\Gamma\left(\frac{\Delta_i+\Delta_j-\Delta_k}{2}\right)\Gamma\left(\frac{\Delta_i+\Delta_k-\Delta_j}{2}\right)\Gamma\left(\frac{\Delta_j+\Delta_k-\Delta_i}{2}\right)}{4\pi^{d/4}\sqrt{2\prod_{n\in\{i,j,k\}}^{3}\Gamma(\Delta_n)\Gamma\left(\Delta_n+1-\frac{d}{2}\right)}}\,\Gamma\left(\frac{\Delta_i+\Delta_j+\Delta_k-d}{2}\right)$$



$$A_2 = \left[\Delta_j\Delta_k + \tfrac{1}{2}(d-\Delta_i-\Delta_j-\Delta_k)(\Delta_j+\Delta_k-\Delta_i)\right]A_1$$

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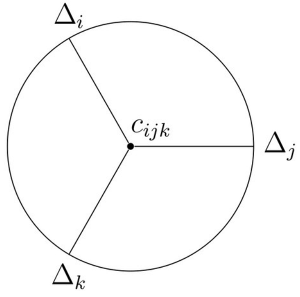

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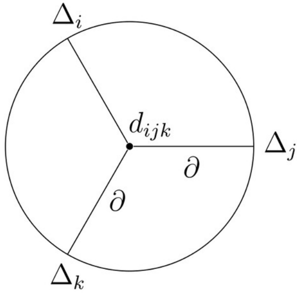

$$A_2 = \left[\Delta_j\Delta_k + \tfrac{1}{2}(d-\Delta_i-\Delta_j-\Delta_k)(\Delta_j+\Delta_k-\Delta_i)\right]A_1$$

→ **3pt-function coefficient computed by**

$$C_{\Phi_i\Phi_j\Phi_k} = c_{ijk}\,A_1 + d_{ijk}\,A_2$$

$$\langle \mathcal{O}_{\Phi_i}(x_1)\mathcal{O}_{\Phi_j}(x_2)\mathcal{O}_{\Phi_k}(x_3)\rangle = \frac{C_{\Phi_i\Phi_j\Phi_k}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}}$$

Bulk cubic couplings in the IR: frame 1

Cubic terms from the potential → $\mathbf{c_{ijk}}$

$$
\begin{aligned}
\mathcal{P}^{(3)} = {} & \frac{9}{4}\left(2\alpha_1\alpha_2\phi_2 + 2\alpha_1\alpha_3\phi_3 - \alpha_2\alpha_3\phi_4\right) - \frac{21}{4}\phi_2\phi_3\phi_4 \\
& + \frac{3}{2\sqrt{2}}\,\widehat{\beta}\left(\alpha_2^2 - \alpha_3^2 - \phi_2^2 + \phi_3^2\right) \\
& + \frac{\sqrt{3}}{8\sqrt{14}}\left(\alpha_2^2 + \alpha_3^2\right)\left(\sqrt{917 + 29\sqrt{7}}\,\rho_1 - \sqrt{917 - 29\sqrt{7}}\,\rho_2\right) \\
& - \frac{\sqrt{3}}{8\sqrt{2}}\left(\phi_2^2 + \phi_3^2\right)\left(\sqrt{371 + 107\sqrt{7}}\,\rho_1 + \sqrt{371 - 107\sqrt{7}}\,\rho_2\right) \\
& + \frac{1}{\sqrt{21}}\,\widehat{\beta}^2\left(\sqrt{217 + 79\sqrt{7}}\,\rho_1 - \sqrt{217 - 79\sqrt{7}}\,\rho_2\right) \\
& - \frac{3}{28}\,\phi_4^2\left(\sqrt{686 + 238\sqrt{7}}\,\rho_1 - \sqrt{686 - 238\sqrt{7}}\,\rho_2\right) \\
& + \frac{\sqrt{3}}{\sqrt{14}}\,\rho_1\rho_2\left(\sqrt{35 + 13\sqrt{7}}\,\rho_1 - \sqrt{35 - 13\sqrt{7}}\,\rho_2\right) \\
& - \frac{\sqrt{3}}{2\sqrt{14}}\,\alpha_1^2\left(\sqrt{2891 - 517\sqrt{7}}\,\rho_1 - \sqrt{2891 + 517\sqrt{7}}\,\rho_2\right) \\
& + \frac{1}{3\sqrt{42}}\left(\sqrt{36155 - 13261\sqrt{7}}\,\rho_1^3 + \sqrt{36155 + 13261\sqrt{7}}\,\rho_2^3\right).
\end{aligned}
$$

And kinetic terms → $\mathbf{d_{ijk}}$

$$
\begin{aligned}
\mathcal{L}^{(3)}_{\text{kin}} = {} & -\left(\phi_2\partial_\mu\alpha_1\partial^\mu\alpha_2 + \phi_3\partial_\mu\alpha_1\partial^\mu\alpha_3 - \phi_4\partial_\mu\alpha_2\partial^\mu\alpha_3\right) \\
& - \left(\phi_2\partial_\mu\alpha_3 + \phi_3\partial_\mu\alpha_2 - \phi_4\partial_\mu\alpha_1\right)\partial^\mu\varphi \\
& + \phi_2\partial_\mu\phi_3\partial^\mu\phi_4 + \phi_3\partial_\mu\phi_2\partial^\mu\phi_4 + \phi_4\partial_\mu\phi_2\partial^\mu\phi_3 \\
& + \frac{1}{2\sqrt{14}}\left(\sqrt{7 - \sqrt{7}}\,\rho_1 + \sqrt{7 + \sqrt{7}}\,\rho_2\right) \\
& \quad \times \left[(\partial_\mu\alpha_1)^2 + (\partial_\mu\alpha_2)^2 + (\partial_\mu\alpha_3)^2 + (\partial_\mu\phi_2)^2 + (\partial_\mu\phi_3)^2 + (\partial_\mu\varphi)^2 + (\partial_\mu\phi_4)^2\right] \\
& + \frac{1}{\sqrt{14}}\left(\phi_2\partial_\mu\phi_2 + \phi_3\partial_\mu\phi_3 + \phi_4\partial_\mu\phi_4\right)\left(\sqrt{7 - \sqrt{7}}\,\partial^\mu\rho_1 + \sqrt{7 + \sqrt{7}}\,\partial^\mu\rho_2\right) \\
& + \frac{\sqrt{3}}{14}\left(\sqrt{7 - \sqrt{7}}\,\rho_1 + \sqrt{7 + \sqrt{7}}\,\rho_2\right)\partial_\mu\rho_1\partial^\mu\rho_2 \\
& + \frac{1}{28}\rho_1\left(\sqrt{35 - 11\sqrt{7}}\,(\partial_\mu\rho_1)^2 + \sqrt{21 + 3\sqrt{7}}\,(\partial_\mu\rho_2)^2\right) \\
& + \frac{1}{28}\rho_2\left(\sqrt{21 - 3\sqrt{7}}\,(\partial_\mu\rho_1)^2 + \sqrt{35 + 11\sqrt{7}}\,(\partial_\mu\rho_2)^2\right).
\end{aligned}
$$

Bulk cubic couplings in the IR: frame 2

Applying change of frame of supergravity couplings using $\Phi_i \partial_\mu \Phi_j \partial^\mu \Phi_k = \frac{m_i^2 - m_j^2 - m_k^2}{2} \Phi_i \Phi_j \Phi_k$

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→ **only c_{ijk}-type terms left:**

$$\begin{aligned}
\mathcal{P}^{(3)}|_{\text{frame 2}} =\ & 3\alpha_1(2\alpha_2\phi_2 + 2\alpha_3\phi_3 - \varphi\phi_4) + \frac{3}{2\sqrt{2}}\widehat{\beta}\left(\alpha_2^2 - \alpha_3^2 - \phi_2^2 + \phi_3^2\right) \\
& + \frac{\sqrt{3}}{2}(\alpha_2^2 + \alpha_3^2 - \phi_2^2 - \phi_3^2)\left(\sqrt{7+\sqrt{7}}\,\rho_1 + \sqrt{7-\sqrt{7}}\,\rho_2\right) \\
& + \frac{1}{\sqrt{21}}\widehat{\beta}^2\left(\sqrt{217+79\sqrt{7}}\,\rho_1 - \sqrt{217-79\sqrt{7}}\,\rho_2\right) \\
& - \frac{\sqrt{3}}{2\sqrt{14}}\phi_4^2\left(\sqrt{77+29\sqrt{7}}\,\rho_1 - \sqrt{77-29\sqrt{7}}\,\rho_2\right) \\
& - \frac{\sqrt{3}}{2\sqrt{14}}\varphi^2\left(\sqrt{35-13\sqrt{7}}\,\rho_1 - \sqrt{35+13\sqrt{7}}\,\rho_2\right) \\
& + \frac{\sqrt{3}}{28}\rho_1\rho_2\left(\sqrt{2065+377\sqrt{7}}\,\rho_1 - \sqrt{2065-377\sqrt{7}}\,\rho_2\right) \\
& - \frac{\sqrt{3}}{2\sqrt{14}}\alpha_1^2\left(\sqrt{3296-523\sqrt{7}}\,\rho_1 - \sqrt{3296+523\sqrt{7}}\,\rho_2\right) \\
& + \frac{1}{84\sqrt{3}}\left(\sqrt{1659343-602999\sqrt{7}}\,\rho_1^3 + \sqrt{1659343+602999\sqrt{7}}\,\rho_2^3\right).
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$\rightarrow$ **only c_{ijk}-type terms left:**

$$\begin{aligned}
\mathcal{P}^{(3)}|_{\text{frame 2}} &= 3\alpha_1(2\alpha_2\phi_2 + 2\alpha_3\phi_3 - \varphi\phi_4) + \frac{3}{2\sqrt{2}}\widehat{\beta}\left(\alpha_2^2 - \alpha_3^2 - \phi_2^2 + \phi_3^2\right) \\
&+ \frac{\sqrt{3}}{2}(\alpha_2^2 + \alpha_3^2 - \phi_2^2 - \phi_3^2)\left(\sqrt{7+\sqrt{7}}\,\rho_1 + \sqrt{7-\sqrt{7}}\,\rho_2\right) \\
&+ \frac{1}{\sqrt{21}}\widehat{\beta}^2\left(\sqrt{217+79\sqrt{7}}\,\rho_1 - \sqrt{217-79\sqrt{7}}\,\rho_2\right) \\
&- \frac{\sqrt{3}}{2\sqrt{14}}\phi_4^2\left(\sqrt{77+29\sqrt{7}}\,\rho_1 - \sqrt{77-29\sqrt{7}}\,\rho_2\right) \\
&- \frac{\sqrt{3}}{2\sqrt{14}}\varphi^2\left(\sqrt{35-13\sqrt{7}}\,\rho_1 - \sqrt{35+13\sqrt{7}}\,\rho_2\right) \\
&+ \frac{\sqrt{3}}{28}\rho_1\rho_2\left(\sqrt{2065+377\sqrt{7}}\,\rho_1 - \sqrt{2065-377\sqrt{7}}\,\rho_2\right) \\
&- \frac{\sqrt{3}}{2\sqrt{14}}\alpha_1^2\left(\sqrt{3296-523\sqrt{7}}\,\rho_1 - \sqrt{3296+523\sqrt{7}}\,\rho_2\right) \\
&+ \frac{1}{84\sqrt{3}}\left(\sqrt{1659343-602999\sqrt{7}}\,\rho_1^3 + \sqrt{1659343+602999\sqrt{7}}\,\rho_2^3\right).
\end{aligned}$$

Note: extremal cubic couplings vanish in frame 2!

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(b) $\langle \mathcal{CCF} \rangle$: $$C_{\alpha_2\alpha_2\widehat{\beta}} = -C_{\alpha_3\alpha_3\widehat{\beta}} = -\frac{3}{4\pi}\,, \quad C_{\phi_2\phi_2\widehat{\beta}} = -C_{\phi_3\phi_3\widehat{\beta}} = \frac{1}{4\pi}\,.$$

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$$C_{\alpha_2\alpha_2\rho_1} = C_{\alpha_3\alpha_3\rho_1} = C_{\alpha_2\phi_2\alpha_1} = C_{\alpha_3\phi_3\alpha_1} = -\frac{\sqrt{3(1+\sqrt{7})}\;\Gamma\left(\frac{2-\sqrt{7}}{2}\right)\Gamma\left(\frac{1+\sqrt{7}}{2}\right)}{2^{\frac{1}{2}+\sqrt{7}}\pi^{\frac{3}{2}}}\,,$$

$$C_{\phi_2\phi_2\rho_1} = C_{\phi_3\phi_3\rho_1} = \frac{7-2\sqrt{7}}{3} \times C_{\alpha_2\alpha_2\rho_1}\,,$$

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$$C_{\phi_4\phi_4\rho_1} = \frac{\sqrt{21(29+11\sqrt{7})}\;\Gamma\left(\frac{5-\sqrt{7}}{2}\right)\Gamma\left(\frac{1+\sqrt{7}}{2}\right)^2\Gamma\left(\frac{3+\sqrt{7}}{2}\right)}{112\pi\,\Gamma(\sqrt{7})}\,,$$

$$C_{\phi_4\phi_4\rho_2} = -\frac{\sqrt{7-2\sqrt{7}}}{2\sqrt{14}} \times C_{\phi_4\phi_4\rho_1}\,,$$

$$C_{\varphi\varphi\rho_1} = \frac{11-4\sqrt{7}}{12} \times C_{\phi_4\phi_4\rho_1}\,,$$

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3pt-functions in the IR: new results for the LS SCFT

One finds 30 non-vanishing 3pt-functions, organised in groups (a)-(e):

(a) $\langle \mathcal{CCC} \rangle$: $C_{\alpha_2\alpha_3\phi_4} = \frac{\gamma}{8\pi}\,, \quad C_{\alpha_2\phi_3\varphi} = C_{\alpha_3\phi_2\varphi} = \frac{\gamma}{16\sqrt{2}\pi}\,, \quad C_{\phi_2\phi_3\phi_4} = 0$

(b) $\langle \mathcal{CCF} \rangle$: $C_{\alpha_2\alpha_2\widehat{\beta}} = -C_{\alpha_3\alpha_3\widehat{\beta}} = -\frac{3}{4\pi}\,, \quad C_{\phi_2\phi_2\widehat{\beta}} = -C_{\phi_3\phi_3\widehat{\beta}} = \frac{1}{4\pi}\,.$

(c) $\langle \mathcal{CCL} \rangle$:

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(d) $\langle \mathcal{FFL} \rangle$:

$$C_{\widehat{\beta}\widehat{\beta}\rho_1} = -\frac{\sqrt{217+79\sqrt{7}}\;\Gamma\big(\frac{3-\sqrt{7}}{2}\big)\Gamma\big(\frac{1+\sqrt{7}}{2}\big)^3}{2\pi\sqrt{42\,\Gamma(\sqrt{7})\Gamma(1+\sqrt{7})}}\,,$$

$$C_{\widehat{\beta}\widehat{\beta}\rho_2} = \frac{\sqrt{91-34\sqrt{7}}}{2\sqrt{42}} \times C_{\widehat{\beta}\widehat{\beta}\rho_1}\,.$$

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(e) $\langle \mathcal{LLL} \rangle$:

$$C_{\rho_1\rho_1\rho_1} = -\frac{\sqrt{1659343-602999\sqrt{7}}\,\Gamma\left(\frac{1+\sqrt{7}}{2}\right)^3\Gamma\left(\frac{-1+3\sqrt{7}}{2}\right)}{56\sqrt{6}\,\pi\left(\Gamma(\sqrt{7})\Gamma(1+\sqrt{7})\right)^{3/2}}\,,$$

$$C_{\rho_1\rho_1\rho_2} = \frac{\sqrt{3(2777257+1042598\sqrt{7})}}{1514\sqrt{14}} \times C_{\rho_1\rho_1\rho_1}\,,$$

$$C_{\rho_1\alpha_1\alpha_1} = \frac{\sqrt{1976297+635680\sqrt{7}}}{1514} \times C_{\rho_1\rho_1\rho_1}\,,$$

$$C_{\rho_1\rho_2\rho_2} = -\frac{4963+1156\sqrt{7}}{42392} \times C_{\rho_1\rho_1\rho_1}\,,$$

$$C_{\rho_2\alpha_1\alpha_1} = \frac{\sqrt{325648561+109746134\sqrt{7}}}{3028\sqrt{42}} \times C_{\rho_1\rho_1\rho_1}\,,$$

$$C_{\rho_2\rho_2\rho_2} = \frac{\sqrt{3(7587462463+2704517498\sqrt{7})}}{84784\sqrt{2}} \times C_{\rho_1\rho_1\rho_1}\,.$$

Summary and Outlook

i) We initialise the study of holographic correlators in a setup beyond (half-)maximal supersymmetry

→ Main new feature: supergravity spectrum contains long multiplets with irrational conformal dimensions

ii) For now, we have new results for 3pt-functions in the LS SCFT

→ Learnt some general lessons for N=1 SCFT's (extremal correlators, exactly marginal deformations)

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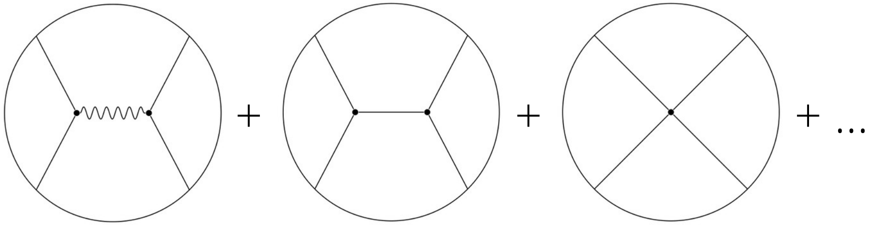
→ Learnt some general lessons for N=1 SCFT's (extremal correlators, exactly marginal deformations)

- Generalisation to all sugra fields (including higher KK modes → efficiently done using ExFT tools) [Duboeuf,Malek,Samtleben'23]
- Next target: 4pt-functions

  → Traditional approach: Witten diagram evaluation

  → Contains new CFT data! How much simplicity remains?



- Other setups:

  → $AdS_5 \times T^{1,1}$ (or more generally $Y^{p,q}$ spaces): spectrum worked out, consistent truncations known
  [Klebanov,Witten'98] [Ceresole,Dall'Agata,D'Auria,Ferrara'99] e.g. [Cassani,Faedo'10]

  → M-theory in AdS_4: richer landscape of AdS_4 vacua, various consistent truncations → [WIP with a student in Leuven]
  e.g. [Bobev,Min,Pilch,Rosso'18]

Backup slide: the full IR spectrum (KK-level n=0)

A total of 128 bosonic and fermionic dof:

	Δ	ϕ	χ	A_μ	$B_{\mu\nu}$	ψ_μ	$h_{\mu\nu}$
$\mathcal{D}(\frac{3}{2},0,0;1)$	$\frac{3}{2}$	$\mathbf{3}_1$					
$\mathrm{Tr}\,\Phi_i\Phi_j$	2		$\mathbf{3}_0$				
complex	$\frac{5}{2}$	$\mathbf{3}_{-1}$					
$\mathcal{D}(2,0,0;0)$	2	$\mathbf{3}_0$					
$\mathrm{Tr}\,\overline{\Phi}T^A\Phi$	$\frac{5}{2}$		$\mathbf{3}_1\oplus\mathbf{3}_{-1}$				
real	3			$\mathbf{3}_0$			
$\mathcal{D}(\frac{9}{4},\frac{1}{2},0;\frac{3}{2})$	$\frac{9}{4}$		$\mathbf{2}_{3/2}$				
$\mathrm{Tr}\,W_\alpha\Phi_j$	$\frac{11}{4}$	$\mathbf{2}_{1/2}$			$\mathbf{2}_{1/2}$		
complex	$\frac{13}{4}$		$\mathbf{2}_{-1/2}$				
$\mathcal{D}(3,0,0;2)$	3	$\mathbf{1}_2$					
$\mathrm{Tr}\,W^\alpha W_\alpha$	$\frac{7}{2}$		$\mathbf{1}_1$				
complex	4	$\mathbf{1}_0$					
$\mathcal{D}(3,\frac{1}{2},\frac{1}{2};0)$	3			$\mathbf{1}_0$			
$J_{\alpha\dot{\alpha}}$	$\frac{7}{2}$					$\mathbf{1}_1\oplus\mathbf{1}_{-1}$	
real	4						$\mathbf{1}_0$

	Δ	ϕ	χ	A_μ	$B_{\mu\nu}$	ψ_μ
$\mathcal{D}(\Delta,0,0;0)$	$1+\sqrt{7}$	$\mathbf{1}_0$				
$\Delta=1+\sqrt{7}$	$\frac{3}{2}+\sqrt{7}$		$\mathbf{1}_1\oplus\mathbf{1}_{-1}$			
K	$2+\sqrt{7}$	$\mathbf{1}_2\oplus\mathbf{1}_{-2}$		$\mathbf{1}_0$		
real	$\frac{5}{2}+\sqrt{7}$		$\mathbf{1}_1\oplus\mathbf{1}_{-1}$			
	$3+\sqrt{7}$	$\mathbf{1}_0$				
$\mathcal{D}(\frac{11}{4},\frac{1}{2},0;\frac{1}{2})$	$\frac{11}{4}$		$\mathbf{2}_{1/2}$			
$\Lambda_{\alpha i}$	$\frac{13}{4}$	$\mathbf{2}_{-1/2}$		$\mathbf{2}_{3/2}$	$\mathbf{2}_{-1/2}$	
complex	$\frac{15}{4}$		$\mathbf{2}_{1/2}\oplus\mathbf{2}_{-3/2}$			$\mathbf{2}_{1/2}$
	$\frac{17}{4}$			$\mathbf{2}_{-1/2}$		
$\mathcal{D}(3,0,\frac{1}{2};\frac{1}{2})$	3		$\mathbf{1}_0$			
Σ_α	$\frac{7}{2}$			$\mathbf{1}_{-1}$	$\mathbf{1}_1$	
complex	4		$\mathbf{1}_{-2}$			$\mathbf{1}_0$
	$\frac{9}{2}$				$\mathbf{1}_{-1}$	