A PATH BETWEEN INTEGRABILITY AND QUANTUM CHAOS

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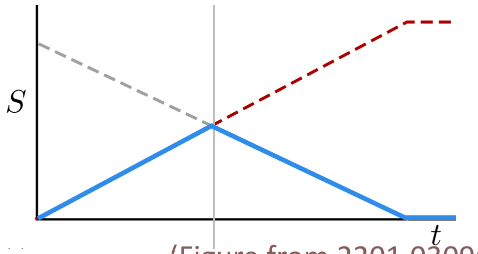
BACKGROUND

BH EVAPORATION AND ENSEMBLE AVERAGE

Penginton; Almheiri, Engelhardt, Marolf, Maxfield;Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini; Penington, Shenker, Stanford, Yang ...

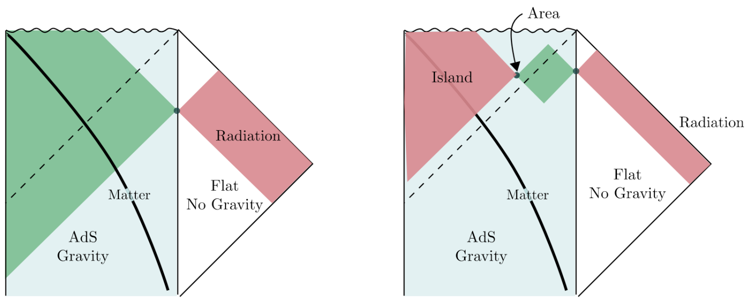
A "solvable" incarnation of the information paradox



(Figure from 2201.03096)

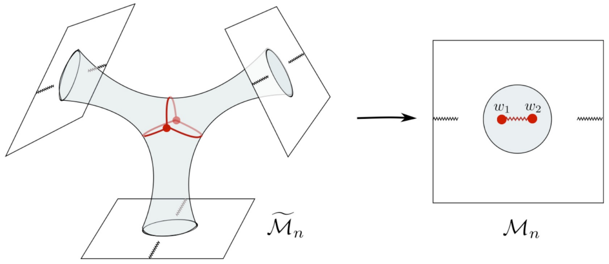
- The information paradox:

  Are Hawking radiations from Blackholes thermal or informative?



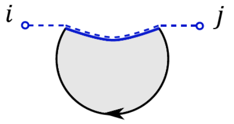
(Figure from 1911.12333)

- Recent breakthroughs in this puzzle in low-dimensional solvable toy models
  - New quantum extremal surface in an evaporating black hole
  - Alternatively, the necessity of including the spacetime wormholes in the gravitational path integral

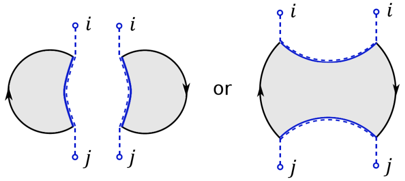


BH EVAPORATION AND ENSEMBLE AVERAGE

- Spacetime wormholes are tied with ensemble averages of theories (Coleman; Giddings Strominger; Maldacena Maoz)
- Evidence including e.g.



$$\overline{\langle\psi_i|\psi_j\rangle} = \delta_{ij},$$



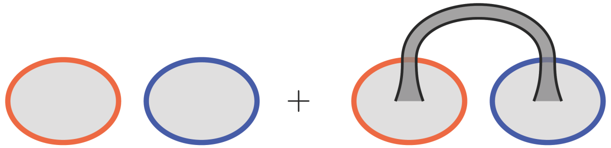
$$\overline{|\langle\psi_i|\psi_j\rangle|^2} = \delta_{ij} + \frac{Z_2}{Z_1^2}$$

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} + x_{ij}, \quad \overline{x_{ij}} = 0, \quad \overline{x_{ij}^2} = \frac{Z_2}{Z_1^2}$$

(Penington, Shenker, Stanford, Yang 2020)

$$\left\langle Z[J_1]\cdots Z[J_n]\right\rangle := \int_{\Phi\sim J} \mathcal{D}\Phi\, e^{-S[\Phi]}$$

$$\left\langle Z[J_1]Z[J_2]\right\rangle =$$



$$\left\langle Z^n\right\rangle = \sum_{p\perp\{1,2,\dots n\}} \lambda^{|p|} = B_n(\lambda) = \sum_{d=0}^{\infty} d^n p_d(\lambda) = \left\langle x^n\right\rangle_{\text{Pois}}, \qquad p_d(\lambda) = e^{-\lambda}\frac{\lambda^d}{d!}$$

(Marolf, Maxfield, 2020
CP, Tian, Yu 2021, **CP**, Tian, Yang 2022)

- Disordered models are special cases of the "ensemble average theories"

HIGH-DIMENSIONAL DISORDERED MODELS

CP, *2018*, Chang, Colin-Ellerin, **CP**, Rangamani, 2021, 2022, and W.I.P.

1. If there exist high dimensional covariant disordered models ?

   2D and 3D models with different numbers of SUSY and tunable parameters

2. Do they share similar nice features as their low dimensional counterparts ?

   Solvable in the large-N limit, analytically in the IR and numerically in general

3. Do they fulfill the usual requirements obeyed by conventional QFTs ?

   Consistent with various **bootstrap** bounds, hence compatible with many requirements

4. If there are clear connections with other well-known conventional QFTs ?

   Observe **higher-spin limits** in different models, which sets up clear connections with higher-spin theories and probably string theory

INTEGRABILITY
&
CHAOS

INTEGRABILITY – CHAOS TRANSITION

- This connection is an example of a transition from chaos to integrability

- There are mainly two types of such transitions in the literature

1. Change some coupling constants directly
   - Turn on some coupling constants from zero
   - Couple an integrable theory with a chaotic theory, and make them compete as we vary some coupling constants

2. Do not change the couplings constants, change other parameters instead

A 2D $\mathcal{N}$=(0,2) MODEL

CP, *JHEP* 12 (2018) 065, Ahn **CP**, JHEP 07 (2019) 092

- $$S = \int d^2z d\theta d\bar{\theta} \left(-\bar{\Phi}^a \partial_{\bar{z}} \Phi^a + \frac{1}{2} \bar{\Lambda}^i \Lambda^i \right) + \int d^2 z d\theta \frac{J_{i a_1 \ldots a_q}}{q!} \Lambda^i \Phi^{a_1} \ldots \Phi^{a_q}$$

  Chiral: $\Phi^a = \phi^a + \sqrt{2}\theta\psi^a + 2\theta\bar{\theta}\partial_z \phi^a \,, \qquad a = 1 \ldots N$

  Fermi: $\Lambda^i = \lambda^i - \sqrt{2}\theta G^i + 2\theta\bar{\theta}\partial_z \lambda^i \,, \qquad i = 1 \ldots M$

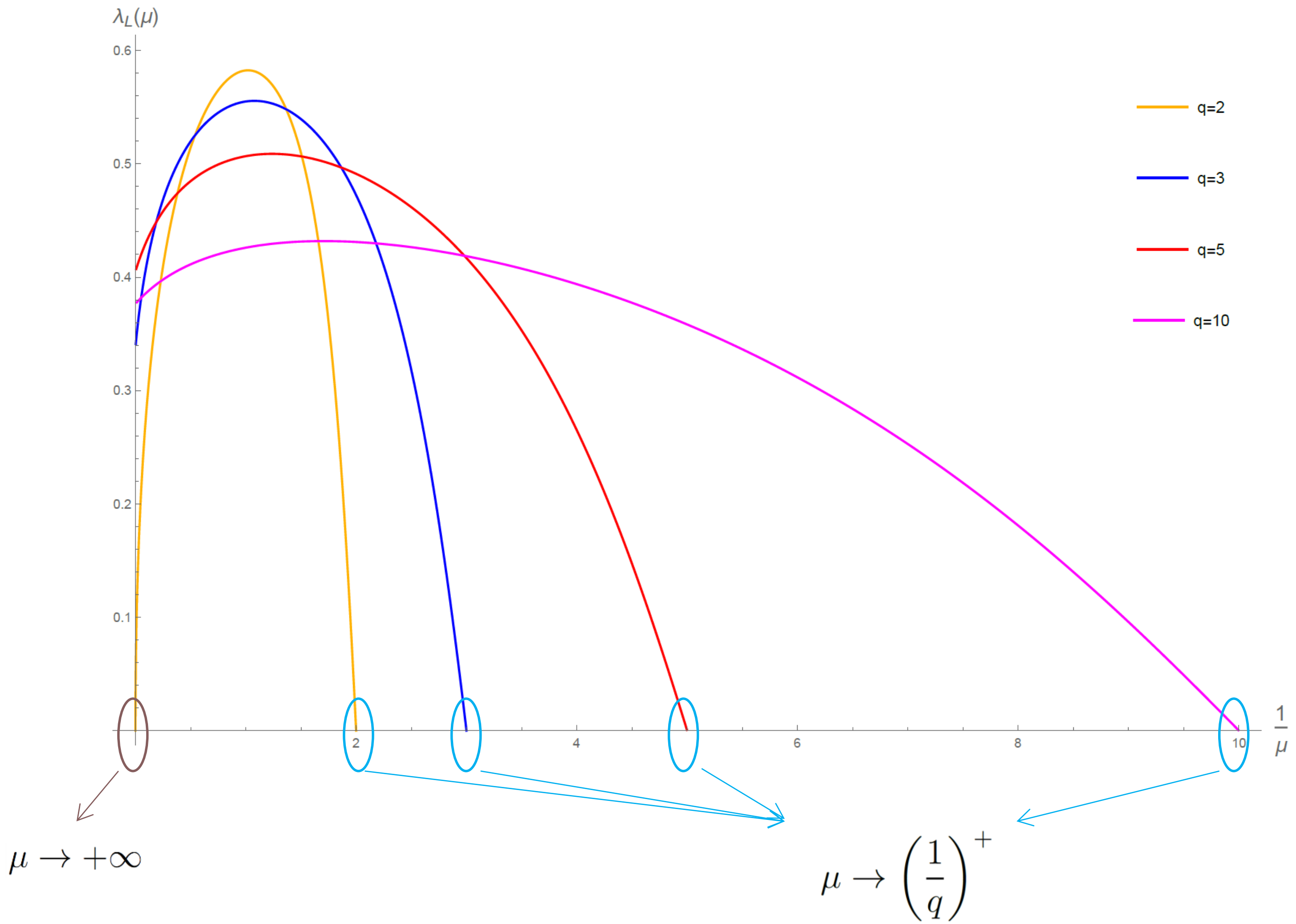
- $N, M \gg 1$, with $\mu = \frac{M}{N}$ fixed (but tunable)

- IR solution $$G_c^I(z_1, z_2) = \frac{n_I}{(z_1 - z_2)^{2h_I} (\bar{z}_1 - \bar{z}_2)^{2\tilde{h}_I}} \qquad I = \phi, \psi, \lambda, G$$

$$h_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}\,, \quad h_\psi = \frac{\mu q^2 + \mu q - 2}{2\mu q^2 - 2}\,, \quad h_\lambda = \frac{q - 1}{2\mu q^2 - 2}\,, \quad h_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}$$

$$\tilde{h}_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}\,, \quad \tilde{h}_\psi = \frac{\mu q - 1}{2\mu q^2 - 2}\,, \quad \tilde{h}_\lambda = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}\,, \quad \tilde{h}_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}\,.$$

TWO INTERESTING LIMITS

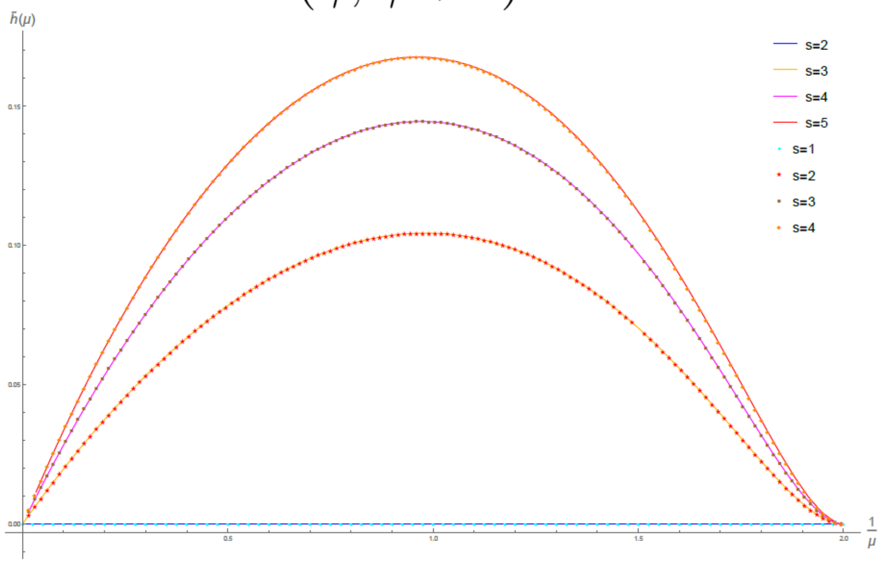


- Lyapunov exponent drops to zero
- "Integrability" takes over ?
- Large symmetries ?
- This happens at fixed J, the transition is due to the screening effect of the interactions

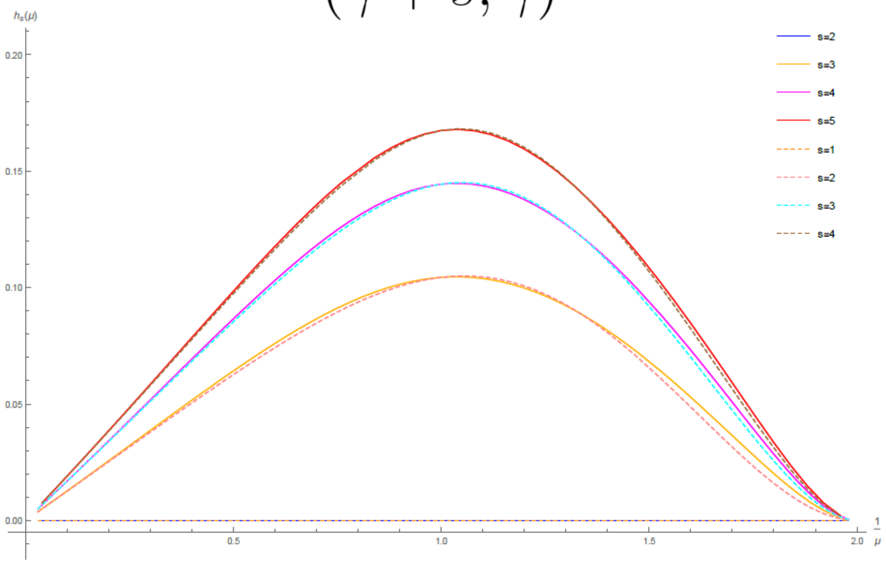
TOWERS OF SPINNING OPERATORS

- A tower of operators has conformal dimension (0,s) or (s,0) in the special limit.
- Emergent higher-spin conserved operators in the two limits!
- Generate large symmetry ➡ nonchaotic

$(\gamma, \gamma + s)$



$(\gamma + s, \gamma)$

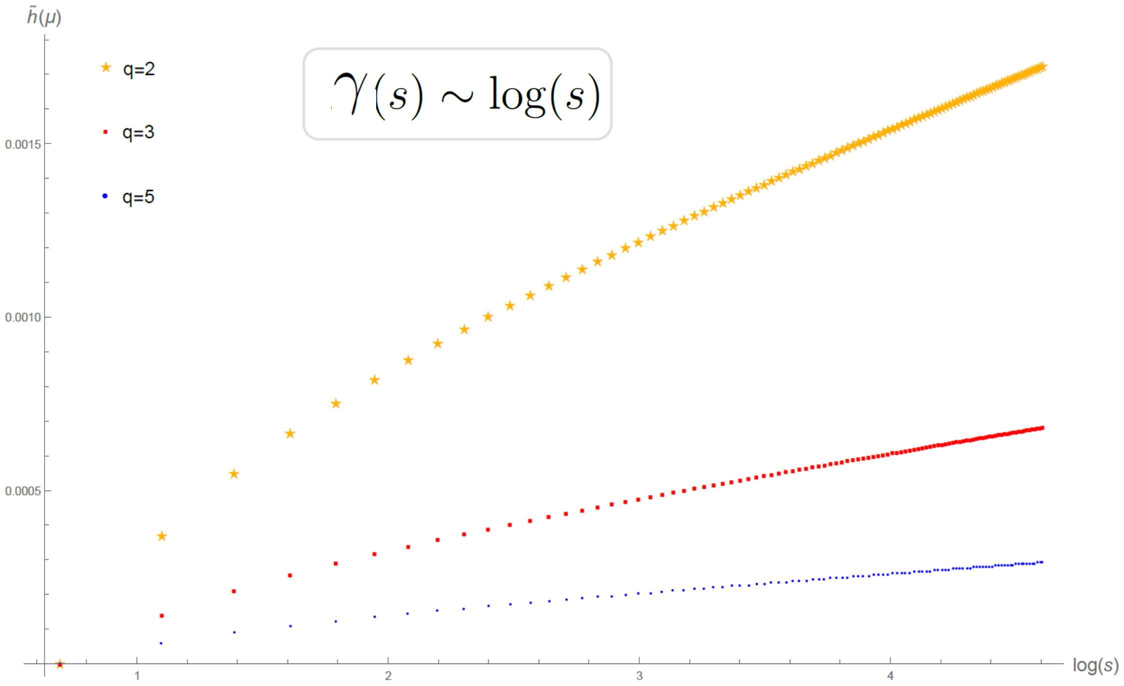


s: spin

γ: anomalous dimension

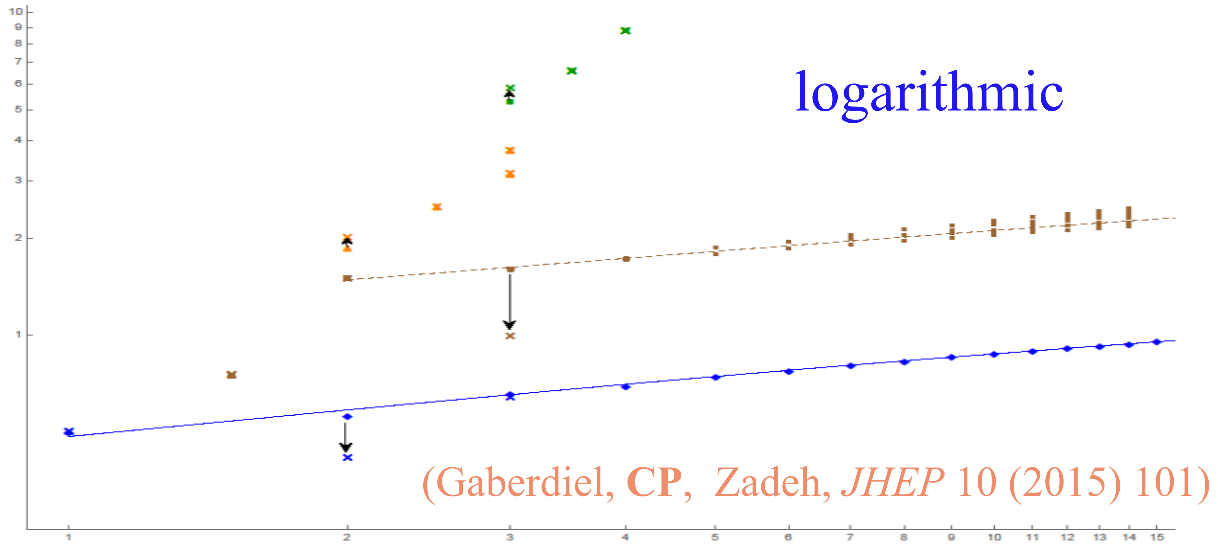
CONNECTIONS WITH CONVENTIONAL THEORIES

- Dispersion relation of this **SYK** model: the anomalous dimension γ logarithmically depends on the spin s



(**CP**, *JHEP* 12 (2018) 065)

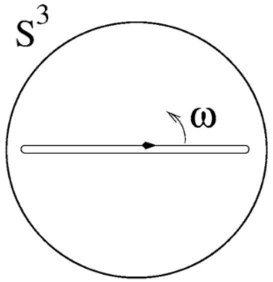
- **Higher-spin** perturbation computation



(Gaberdiel, **CP**, Zadeh, *JHEP* 10 (2015) 101)

- Rotating folded closed long **string** in AdS

$$E - S = \frac{\sqrt{\lambda}}{\pi} \ln(S/\sqrt{\lambda}) + \cdots \qquad \lambda = g_{\mathrm{YM}}^2 N$$



logarithmic due to the AdS geometry

(Gubser, Klebanov, Polyakov, *Nucl.Phys.B* 636 (2002) 99-114)

A 3D DISORDERED MODELS

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129, 011603

- We can in fact consider more general 2+1d disordered models
  - ✓ with supersymmetry

$$L_{\text{susy}} = -\int d^2\theta d^2\bar{\theta}\left(\bar{\mathcal{p}}_i(y^+)\,\mathcal{p}^i(y) + \bar{\mathcal{s}}_a(y^+)\,\mathcal{s}^a(y)\right) - \left[\int d^2\theta \frac{1}{2} g_{aij}\,\mathcal{s}^a(y)\,\mathcal{p}^i(y)\mathcal{p}^j(y) + \text{c.c}\right]$$

where $\mathcal{p}^i$ and $\mathcal{s}^a$ are chiral $\mathcal{N}$=2 multiplets with $i = 1\ldots N$ and $a = 1\ldots M$

  - ✓ or without supersymmetry

$$L_{\text{bos}} = \frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi^i + \frac{1}{2} g_{aij}\sigma^a \phi^i \phi^j - \frac{1}{4}(\sigma^a)^2$$

where ϕ_i and σ^a are bosonic fields with $i = 1\ldots N$ and $a = 1\ldots M$

- Can solve the model in the large-N limit in the IR analytically

$$N \to \infty, \qquad \lambda \equiv M/N, \quad \text{fixed}$$

3D DISORDERED MODELS: HIGHER-SPIN LIMITS

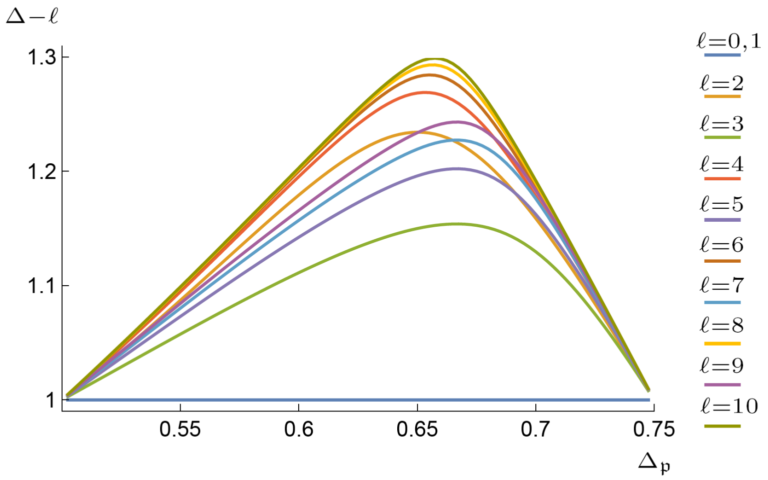
Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- Most of the details of the model are quite different from the 0+1d and 1+1d models

- Nevertheless there is again a clear connection to higher-spin theories

- There are special limits

  - Free p $\quad \lambda \to 0, \quad \Delta_p = \frac{1}{2}, \quad \Delta_s = 1$ higher-spin
  - Free s $\quad \lambda \to \infty, \quad \Delta_p = \frac{3}{4}, \quad \Delta_s = \frac{1}{2}$ higher-spin
  - Diagonal $\quad \lambda \to \frac{1}{2}, \quad \Delta_p = \frac{2}{3}, \quad \Delta_s = \frac{2}{3}$ the 2+1D SYK model



- This indicates the connection to higher-spin theories is probably universal

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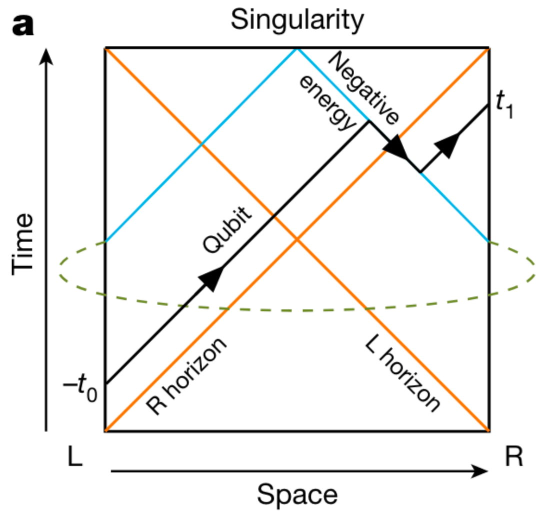
nature

Article | Published: 30 November 2022

Traversable wormhole dynamics on a quantum processor

Daniel Jafferis, Alexander Zlokapa, Joseph D. Lykken, David K. Kolchmeyer, Samantha I. Davis, Nikolai Lauk, Hartmut Neven & Maria Spiropulu

Nature **612**, 51–55 (2022) | Cite this article



Gao-Jafferis-Wall JHEP 12 (2017)

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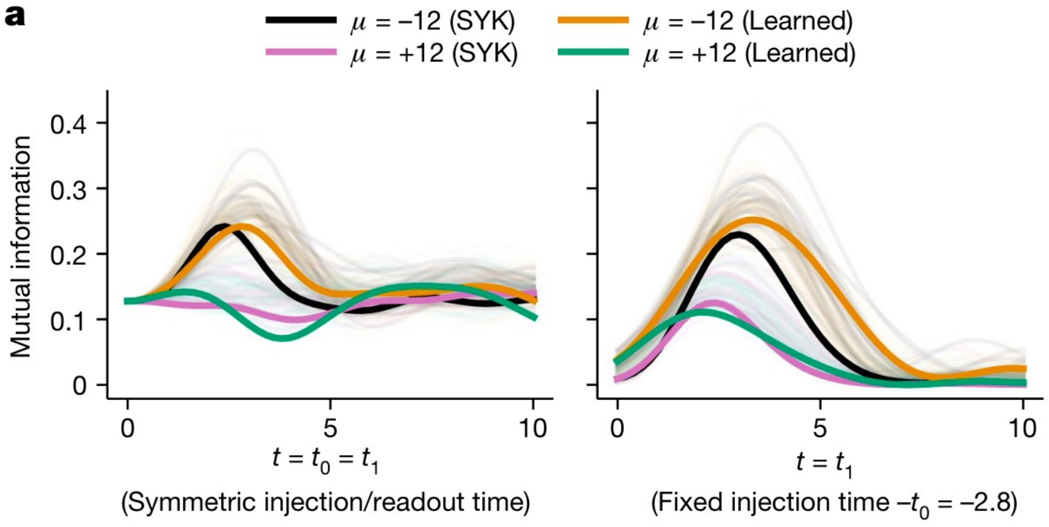

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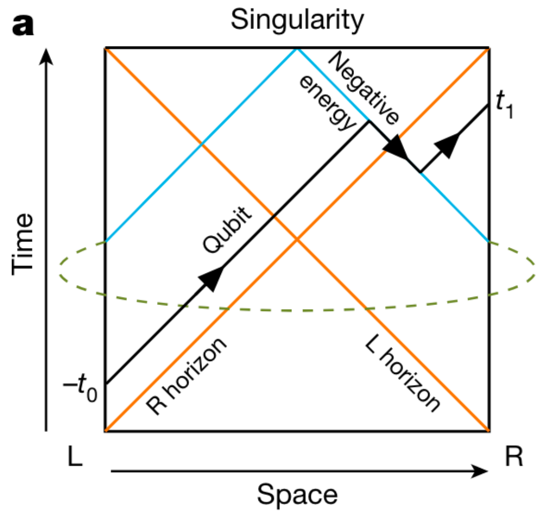
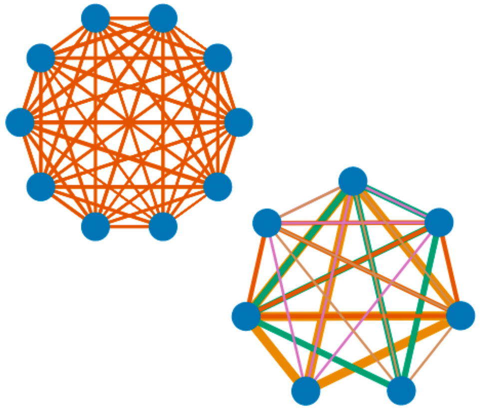


SYK model

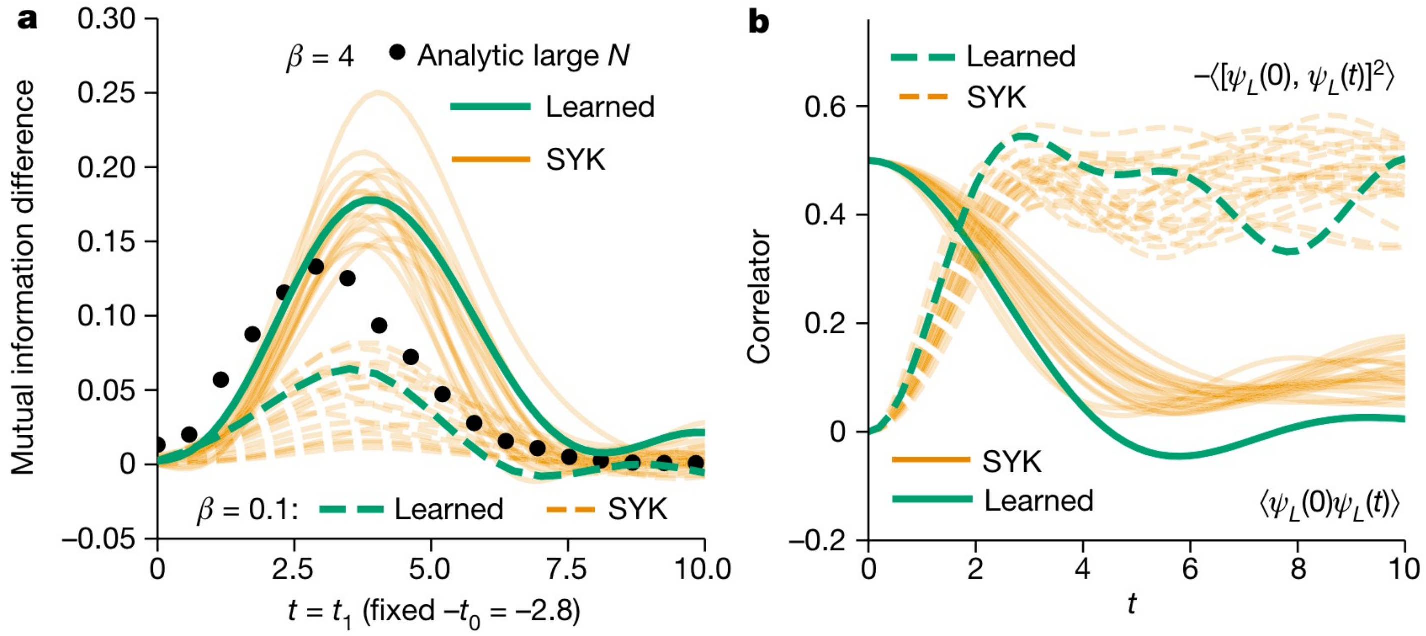


$$|\mathrm{TFD}\rangle = \frac{1}{Z}\sum_n e^{-\beta E_n/2}\,|n\rangle_L\,|n\rangle_R$$

$$H = H_L + H_R$$

TRAVERSABLE WORMHOLES ?

$$H_{\mathrm{L,R}} = -0.36\psi^1\psi^2\psi^4\psi^5 + 0.19\psi^1\psi^3\psi^4\psi^7 - 0.71\psi^1\psi^3\psi^5\psi^6 + 0.22\psi^2\psi^3\psi^4\psi^6 + 0.49\psi^2\psi^3\psi^5\psi^7,$$



TRAVERSABLE WORMHOLES ?

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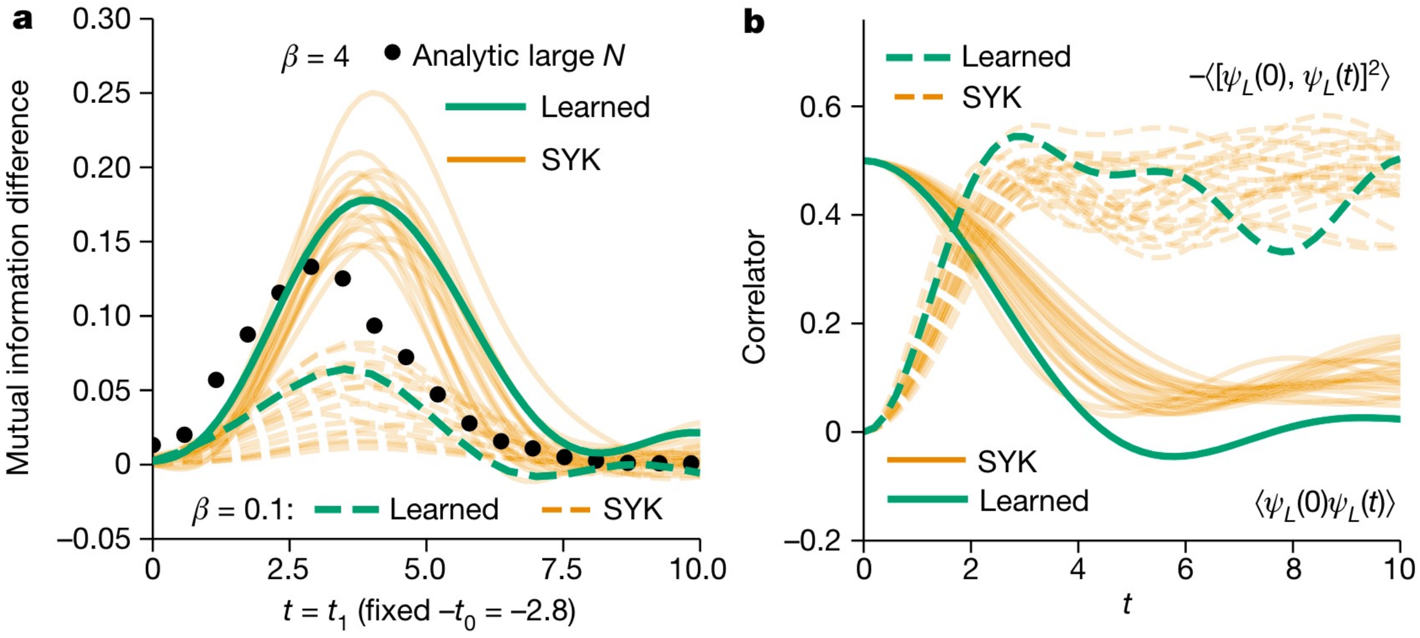


Wait, isn't H commuting?

Namely there are lots of conserved quantities?

So shouldn't the theory Be integrable?

But gravity is chaotic …

COMMUTING SYK

Gao JHEP 01 (2024)

- Terms in the Hamiltonian commute
- A more general representation of such models

$$H = \sum_{i_k} \mathcal{J}_{i_1 \cdots i_{q/2}} X_{i_1} \cdots X_{i_{q/2}} \qquad X_i = \psi_{2i-1}\psi_{2i}$$

- Also known as the Sherrington-Kirkpatrick (SK) model for q=4
  - Integrable, should not be chaotic
  - The partition function is Gaussian, not dense near the edge, not holographic

$$\overline{Z} = \int dE e^{-\beta E} \rho(E) \implies \rho(E) = \frac{1}{\mathcal{J}} \sqrt{\frac{q}{\pi N}} \exp(-qE^2/(N\mathcal{J}^2))$$

THE COMMUTING SYK IS EASIER TO REALIZE ON QUANTUM COMPUTERS, AS IN THE NATURE PAPER.

HOW CAN WE MAKE FURTHER USE OF IT ?

MORE IS DIFFERENT ?

Gao, Lin, CP, WIP

- We can consider the following alternative model

$$\tilde{H} = \frac{1}{\sqrt{d}} \sum_{a=1}^{d} \tilde{H}_a, \quad \tilde{H}_a = \sum_{i_1 < \cdots < i_p} J^a_{i_1 \cdots i_p} \mathcal{X}^a_{i_1} \cdots \mathcal{X}^a_{i_p} ;$$

$$\mathcal{X}^a_j \equiv i\psi_{2j-1}\psi_{(2j-2+2a)_{2N}}$$

- d = 1, the original commuting SYK

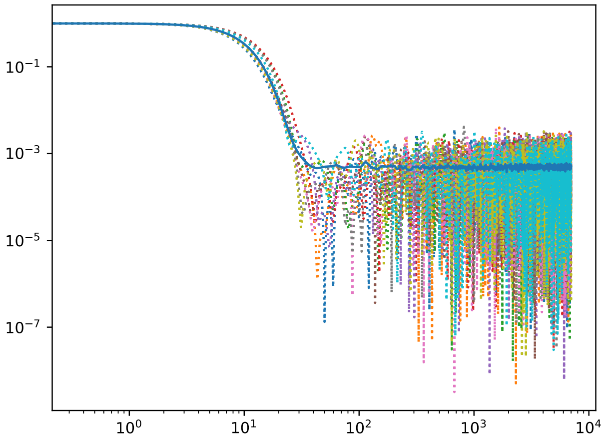
- d > 1, the model is not commuting, but similar enough

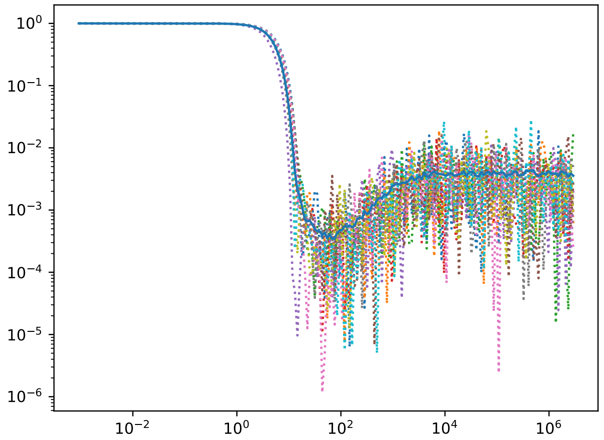
- Is this model better (holographic) ?

MORE IS DIFFERENT !

- Spectral form factor

  Ramp and plateau for the d>1 case

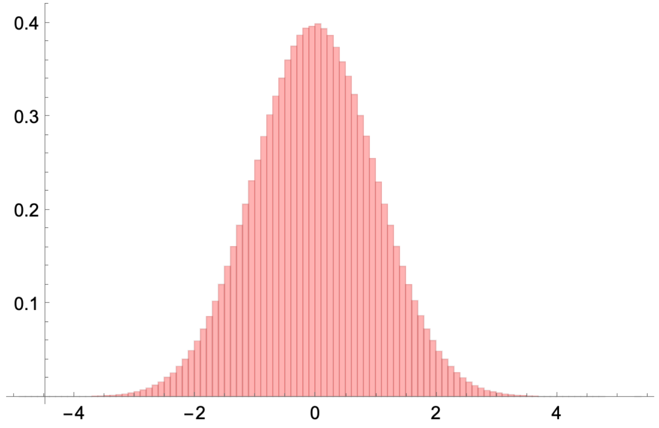


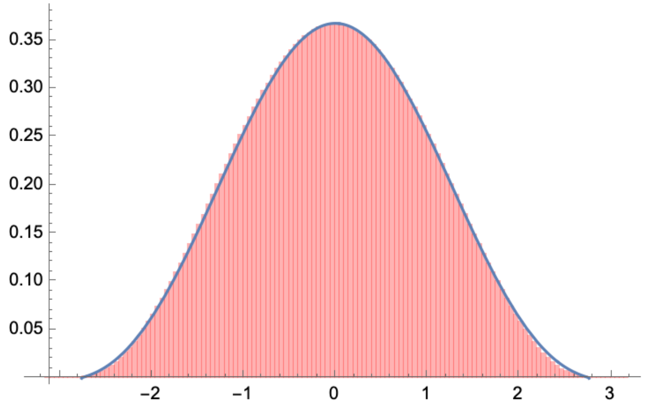


- Spectrum

  d = 1, Gaussian

  d > 1, ~ regular SYK



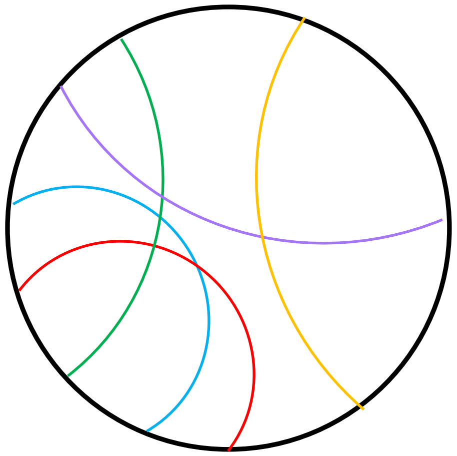


$d \to \infty: \quad \cong$ REGULAR SYK

- We can first take the extremal limit $d \to \infty$
- Solve the model in the double scaling limit

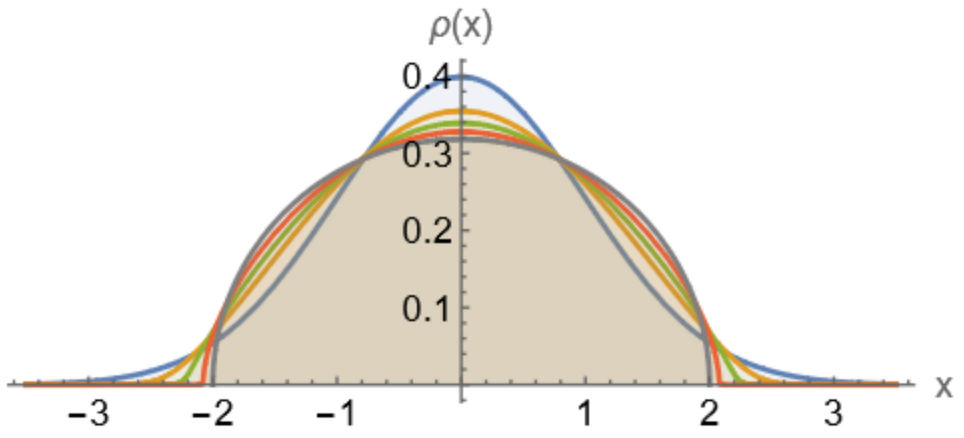
$$N \to \infty, \qquad p \to \infty, \qquad \lambda = \frac{4p^2}{N}$$

- Chord diagrams, each intersections contributes a factor of $q = e^{-\lambda}$
- A typical chord diagram: all chords are different in color $\lim_{d \to \infty} \binom{d}{n} n!/d^n = 1$
- The contributions are the same as those in regular SYK



GENERAL d , $q \to 0$

- Small d>1, but more complicated
- Again try to solve these models in the double scaling limit
- One solvable limit is $q \to 0$
- Different color crossing is forbidden,
  only same color crossing is allowed
- The model can be solved using free probability



THANK YOU!

A 3D SYK MODEL

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- It turns out possible to construct an $\mathcal{N}$=2 Supersymmetry SYK model

$$\mathrm{L} = -\int d^2\theta d^2\bar{\theta}\left(\bar{\Phi}_i(y^\dagger)\Phi_i(y)\right) - \left[\int d^2\theta \frac{1}{3} g_{ijk}\Phi_i(y)\Phi_j(y)\Phi_k(y) + \mathrm{c.c}\right]$$

$$\mathrm{P}(g_{ijk}) \propto e^{-N^2 \frac{g_{ijk}\bar{g}_{ijk}}{J}}, \qquad \langle g_{ijk}\rangle = 0, \qquad \langle g_{ijk}\bar{g}_{ijk}\rangle = \frac{J}{N^2}.$$

with N flavors of chiral multiplets

$$\Phi(X) = \phi(y) + \sqrt{2}\theta^\alpha \psi_\alpha(y) + \theta^2 F(y) \qquad \bar{\Phi}(X^\dagger) = \bar{\phi}(y^\dagger) + \sqrt{2}\bar{\theta}^\alpha \bar{\psi}_\alpha(y^\dagger) + \bar{\theta}^2 \bar{F}(y^+)$$

- In components:

$$\mathrm{L} = -i\bar{\psi}_i \not{\partial}\psi_i + \partial_\mu\bar{\phi}_i\partial_\mu\phi_i - \bar{F}_i F_i - g_{ijk}\left(\phi_i\phi_j F_k - \psi_i\psi_j\phi_k\right) - \bar{g}_{ijk}\left(\bar{\phi}_i\bar{\phi}_j\bar{F}_k - \bar{\psi}_i\bar{\psi}_j\bar{\phi}_k\right)$$

- The model is again solvable, and its properties indicates that the disordered theory flow to a normal IR fixed point that has no obvious difference from the other conventional models.

SPECTRUM: THE 3D SYK MODEL v.s. $\mathcal{N}$=2 BOOTSTRAP

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- The IR spectrum is within the bounds obtained from numerical bootstrap

Operators	ℓ	Δ	Bootstrap bound
$(\bar{\Phi}\Phi)$	0	1.6994	<1.9098
$(\bar{\Phi}\Phi)'$	0	3.4295	<5.3
J'	1	4.2676	<5.25

Bobev, El-Showk, Mazac, Paulos, *Phys. Rev. Lett.* 115 (2015) 051601

- Anomalous dimension $\tau = \Delta - \ell = 2\Delta_\phi + 2m + \gamma(m,\ell)$

The large-spin limit, ie fixed *m*, large ℓ limit

$$\gamma(m,\ell) = (-1)^{\ell+1}\frac{g_3(\Delta_\phi)}{\ell^{\Delta_\phi}}\frac{\Gamma(m-\Delta_\phi+1)}{\Gamma(m+1)}, \qquad \ell \gg 1$$

agrees with results from the light-cone analytic bootstrap

$$\gamma(m,\ell) = (-1)^{\ell}\frac{C_m}{\ell^{\tau_{\min}}} \qquad \tau_{\min} = \tau_\phi = \Delta_\phi$$

Fitzpatrick, Kaplan, Poland, Simmons-Duffin, *JHEP* 12 (2013) 004

3D DISORDERED MODELS: OTHER PROPERTIES

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- The anomalous dimensions

$$\lim_{\ell\gg 1}\gamma_{\phi,\sigma}(\ell,n)\sim\frac{1}{\ell^{2\Delta_\phi}},\quad \lim_{n\gg 1}\gamma_{\phi,\sigma}(\ell,n)\sim\frac{1}{n^{4\Delta_\phi}}$$

again agrees with the bootstrap results

- The central charges behaves as normal field theories in the special limits

$$C_T\to N\left(\frac{3}{2}-\frac{20}{3\pi^2}\lambda+\cdots\right)\qquad \text{as}\qquad \lambda\to 0$$

- All these properties indicates that the disordered theory flow to a normal IR fixed point that has no obvious difference from the other conventional models.

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$$\Phi(X) = \phi(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + \theta^2 F(y) \qquad \bar{\Phi}(X^\dagger) = \bar{\phi}(y^\dagger) + \sqrt{2}\bar{\theta}^\alpha\bar{\psi}_\alpha(y^\dagger) + \bar{\theta}^2\bar{F}(y^+)$$

- In components:

$$\mathrm{L} = -i\bar{\psi}_i \not{\partial}\psi_i + \partial_\mu\bar{\phi}_i\partial_\mu\phi_i - \bar{F}_i F_i - g_{ijk}\left(\phi_i\phi_j F_k - \psi_i\psi_j\phi_k\right) - \bar{g}_{ijk}\left(\bar{\phi}_i\bar{\phi}_j\bar{F}_k - \bar{\psi}_i\bar{\psi}_j\bar{\phi}_k\right)$$

- The model is again solvable, and its properties indicates that the disordered theory flow to a normal IR fixed point that has no obvious difference from the other conventional models.

SPECTRUM: THE 3D SYK MODEL v.s. $\mathcal{N}$=2 BOOTSTRAP

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- The IR spectrum is within the bounds obtained from numerical bootstrap

Operators	ℓ	Δ	Bootstrap bound
$(\bar{\Phi}\Phi)$	0	1.6994	<1.9098
$(\bar{\Phi}\Phi)'$	0	3.4295	<5.3
J'	1	4.2676	<5.25

Bobev, El-Showk, Mazac, Paulos, *Phys. Rev. Lett.* 115 (2015) 051601

- Anomalous dimension $\tau = \Delta - \ell = 2\Delta_\phi + 2m + \gamma(m,\ell)$

The large-spin limit, ie fixed m, large ℓ limit

$$\gamma(m,\ell) = (-1)^{\ell+1}\frac{g_3(\Delta_\phi)}{\ell^{\Delta_\phi}}\frac{\Gamma(m-\Delta_\phi+1)}{\Gamma(m+1)}, \qquad \ell \gg 1$$

agrees with results from the light-cone analytic bootstrap

$$\gamma(m,\ell) = (-1)^{\ell}\frac{C_m}{\ell^{\tau_{\min}}} \qquad \tau_{\min} = \tau_\phi = \Delta_\phi$$

Fitzpatrick, Kaplan, Poland, Simmons-Duffin, *JHEP* 12 (2013) 004

3D DISORDERED MODELS: OTHER PROPERTIES

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- The anomalous dimensions

$$\lim_{\ell\gg 1}\gamma_{\phi,\sigma}(\ell,n)\sim\frac{1}{\ell^{2\Delta_\phi}},\quad \lim_{n\gg 1}\gamma_{\phi,\sigma}(\ell,n)\sim\frac{1}{n^{4\Delta_\phi}}$$

again agrees with the bootstrap results

- The central charges behaves as normal field theories in the special limits

$$C_T\to N\left(\frac{3}{2}-\frac{20}{3\pi^2}\lambda+\cdots\right)\qquad \text{as}\qquad \lambda\to 0$$

- All these properties indicates that the disordered theory flow to a normal IR fixed point that has no obvious difference from the other conventional models.